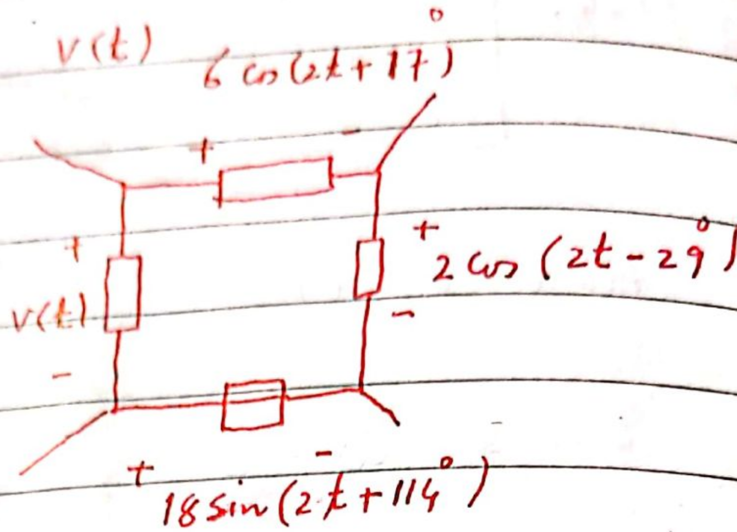
1. Find $v(t)$

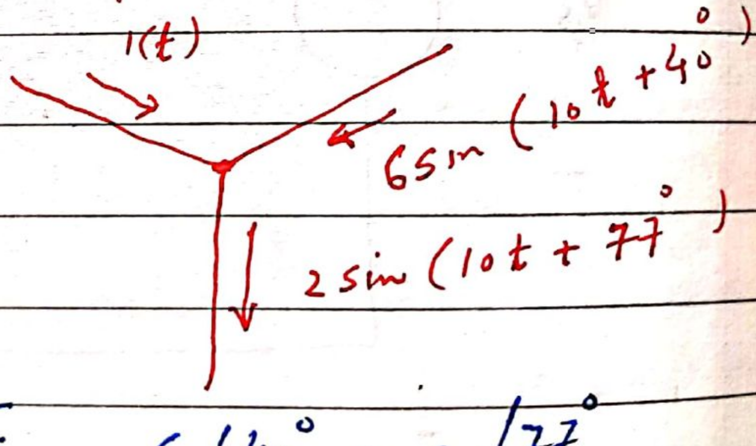


$$\bar{V} = 6\angle 17^\circ + 2\angle -29^\circ - 18\angle 114^\circ$$

$$\bar{V} = 11.09\angle -143.9^\circ$$

$$\therefore v(t) = 11.09\cos(2t - 143.9^\circ)$$

2. Find $i(t)$



$$\bar{I} = -6\angle 40^\circ + 2\angle 77^\circ$$

$$\bar{I} = 4.56\angle 114.7^\circ + 90^\circ$$

$$= 4.56\sin(10t + 204.7^\circ)$$

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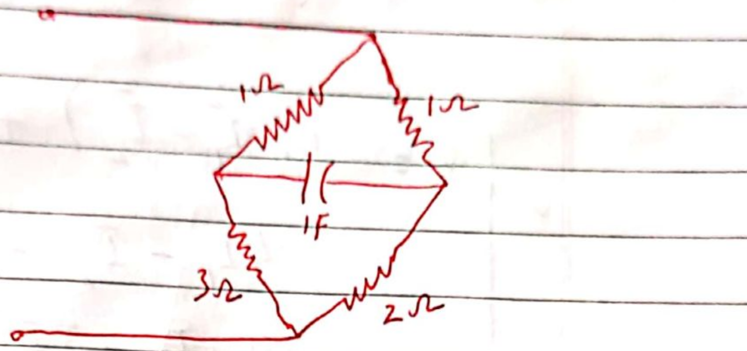
or $\bar{I}_1 = -8\angle{-50°} + 2\angle{-13°}$

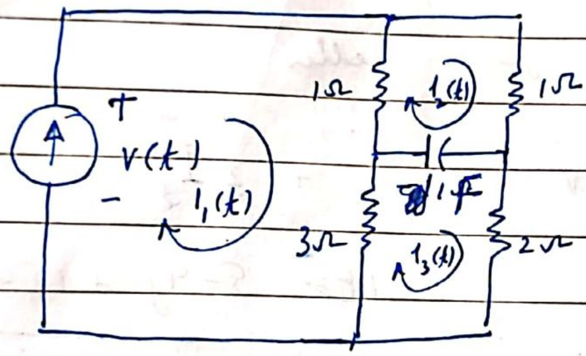
$= 4.56\angle{114.7°}\ A$

$\therefore i(t) = 4.56 \cos(10t + 114°.7)A$

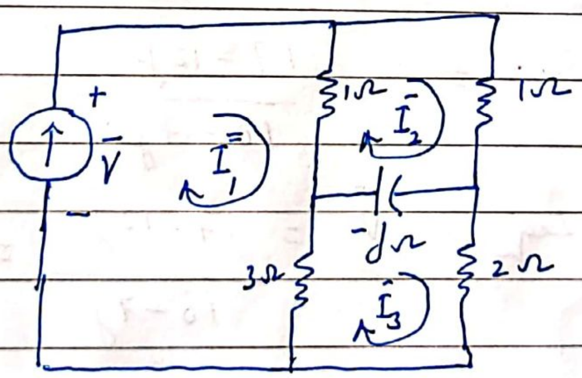
(3) What is the impedance looking into this two-terminal network at $\omega = 1$ rad/s?



1 sin t



$1\angle{0°}$



$\bar{I}_1 = 1\angle{0°}$

$-\bar{I}_1 + (2-j)\bar{I}_2 + j\bar{I}_3 = 0$

$(2-j)\bar{I}_2 + j\bar{I}_3 = 1\angle{0°}$ —— (1)

$$-3\bar{I_1} + j\bar{I_2} + (5-j)\bar{I_3} = 0$$

$$j\bar{I_2} + (5-j)\bar{I_3} = 3\angle 0^\circ$$

$$\bar{I_2} = \left(\frac{5-4j}{10-7j}\right), \quad \bar{I_3} = \left(\frac{6-4j}{10-7j}\right)$$

$$\bar{V} = 1(\bar{I_1} - \bar{I_2}) + 3(\bar{I_1} - \bar{I_3})$$

$$= 4\bar{I_1} - \bar{I_2} - 3\bar{I_3}$$

$$= 4 - \bar{I_2} - 3\bar{I_3}$$

or better

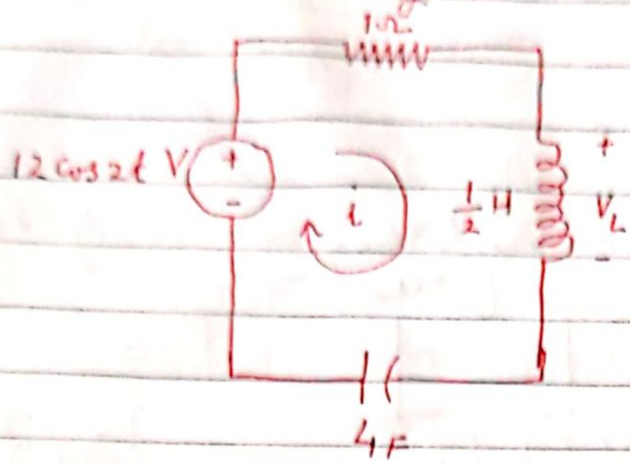
$$\bar{V} = \bar{I_2} + 2\bar{I_3}$$

$$= \frac{5-4j+12-8j}{10-7j}$$

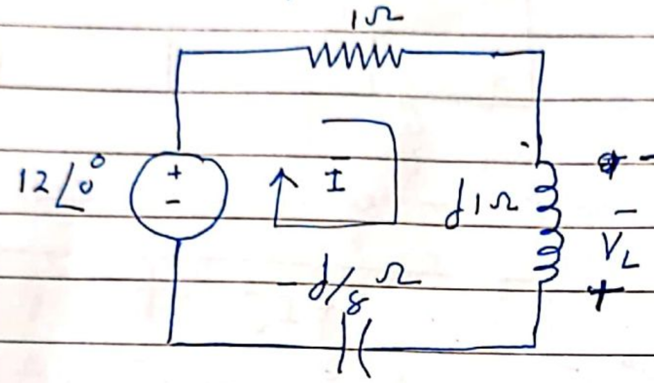
$$= \frac{17-12j}{10-7j}$$

$$\therefore Z_{eq} = \bar{V} = \frac{17-12j}{10-7j} = \left(\frac{254-j}{149}\right)\Omega$$

(4) Find ac steady-state value of $v_L(t)$ and $i(t)$



Phasor equivalent ckt. :



$$\bar{V_L} = \frac{-j}{1+j\left(1-\frac{1}{8}\right)} \times 12\angle 0^\circ$$

$$= \frac{-j}{1+j\frac{7}{8}} \times 12\angle 0^\circ$$

$$= \frac{8j}{8+j7} \times 12\angle -90^\circ$$

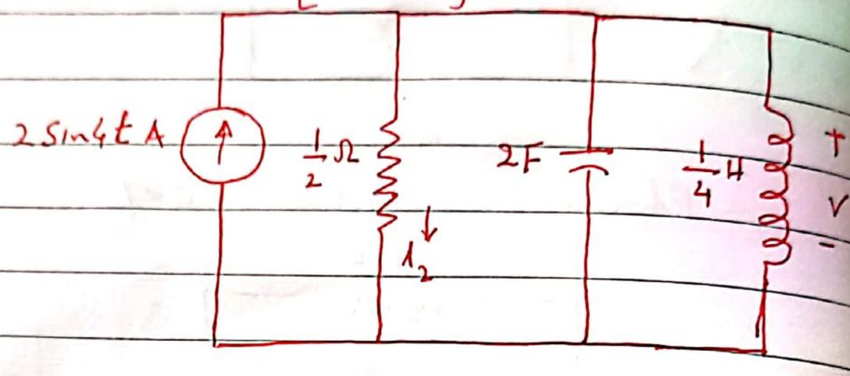
$$= \frac{96\angle -90^\circ}{\sqrt{64+49}\angle \tan^{-1}\frac{7}{8}}$$

$$= 9.03\angle -131.2^\circ$$

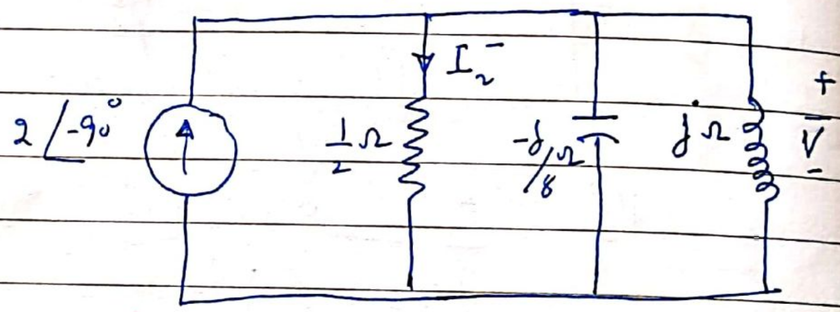
$$\therefore v_L(t) = 9.03\cos(2t - 131.2^\circ)$$

$$\bar{I} = \frac{12\angle 0^\circ}{1+j7/8} = 9.03\angle -41.12^\circ$$

(5) Find $\left\{ i_r(t) \text{ \& } v(t) \right\}$ in steady state



Phasor equivalent ckt. :



$$\bar{I}_2 = \frac{1/\tfrac{1}{2}}{2+j8-j} \times 2\angle -90^\circ$$

$$= \left(\frac{4}{2+j7}\right)\angle -90^\circ$$

$$= 0.55\angle -164.1^\circ$$

$$\therefore \bar{V} = \frac{\bar{I}_2}{2} = 0.275\angle -164.1^\circ$$

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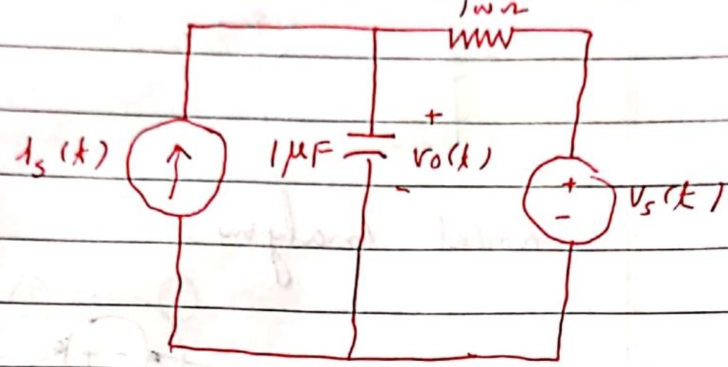
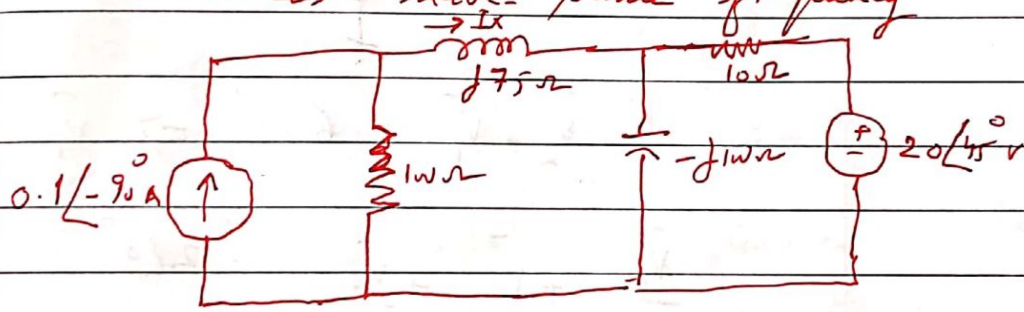
$\therefore\ i_2(t) = 0.55 \cos(4t - 164.1^\circ)$
and $V(t) = 0.27 \cos(4t - 164.1^\circ)$

(6) Use superposition to find the output voltage $v_0(t)$ in the following circuit if $i_s(t) = 100 \cos(10{,}000t)$ mA and $V_s(t) = 20 \cos(20{,}000t - 45^\circ)$



$V_0(t) = 7.07 \cos(10kt - 45^\circ) + 8.94 \cos(20kt - 108.4^\circ)$ V

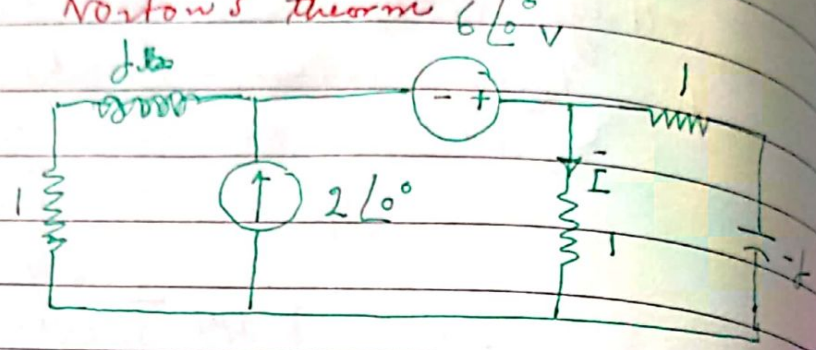
(7) Find the phasor current $\bar{I}_x$
Two Sources have same frequency



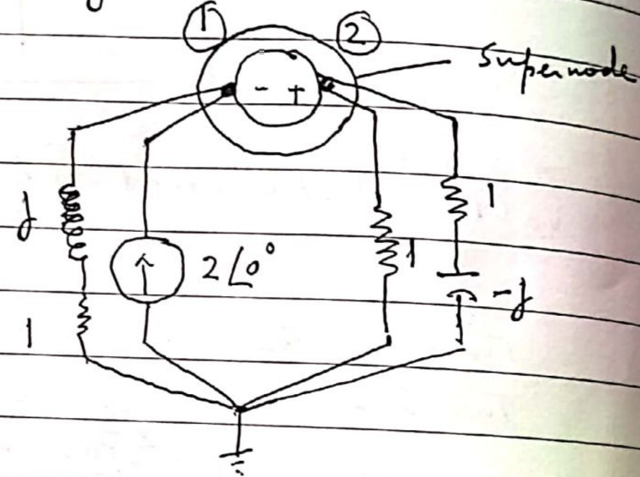
$\bar{I}_x = 0.206 \angle -158^\circ$ A

(8) Find current $\bar{I}$ using:

1) Nodal Analysis
2) Mesh Analysis
3) Superposition theorm
4) Thevenin's theorm
5) Norton's theorm



Nodal Analysis:



$$\bar{V}_2 - \bar{V}_1 = 6\angle 0^\circ \Rightarrow \bar{V}_1 = \bar{V}_2 - 6\angle 0^\circ$$

$$\frac{\bar{V}_1}{j+1} + \frac{\bar{V}_2}{1} + \frac{\bar{V}_2}{1-j} = 2\angle 0^\circ$$

$$\bar{V}_2\left[1 + \frac{1}{1-j}\right] + \frac{\bar{V}_2 - 6\angle 0^\circ}{j+1} = 2\angle 0^\circ$$

$$\bar{V}_2\left[\frac{1-j+1}{1-j}\right] + \frac{\bar{V}_2 - 6}{1+j} = 2$$

$$V_2\left[\frac{2-j}{2}\right](1+j) + \frac{\bar{V}_2 - 6}{2}(1-j) = 2$$

$$\bar{V}_2[2+j-j+1] + \bar{V}_2 - 6 + 6j = 4$$

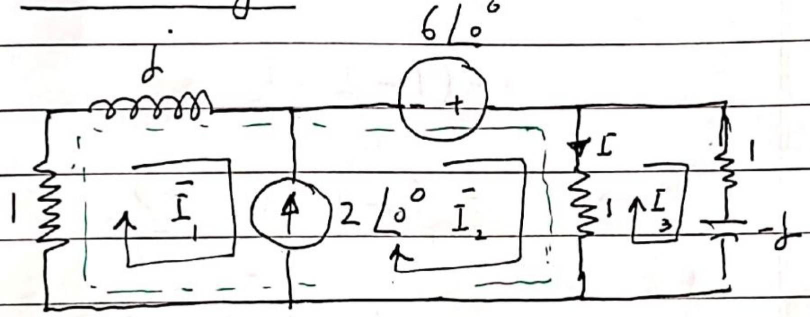
$$3\bar{V}_2 + \bar{V}_2 = 10-6j$$

$$4\bar{V}_2 = 10-6j$$

$$\bar{V}_2 = \frac{5-3j}{2}$$

$$\bar{I} = \frac{5-3j}{2}$$

Mesh Analysis:



$$-\bar{I}_1 + \bar{I}_2 = 2\angle 0^\circ$$ Constraint equation

$$-\bar{I}_2 + (2-j)\bar{I}_3 = 0$$ essential loop

$$(1+j)\bar{I}_1 + (\bar{I}_2 - \bar{I}_3) = 6\angle 0^\circ$$ Superloop

$$\bar{I}_1 = \bar{I}_2 - 2\angle 0^\circ$$

$$-\bar{I}_2 + (2-j)I_3 = 0 \quad \text{— (1)}$$

$$(1+j)[\bar{I}_2 - 2] + (\bar{I}_2 - \bar{I}_3) = 6$$

$$\bar{I}_2 - 2 + j\bar{I}_2 - 2j + \bar{I}_2 - \bar{I}_3 = 6$$

$$(2+j)\bar{I}_2 - \bar{I}_3 = 8+2j \quad \text{— (ii)}$$

$$\downarrow$$

$$(2+j)((2-j)\bar{I}_3) - \bar{I}_3 = 8+2j$$

$$\cancel{4-2j\bar{I}_3+2j+\bar{I}_3-\bar{I}_3=8+2j}$$

$$\cancel{-2j\bar{I}_3 = 4-2j}$$

$$(4+1)\bar{I}_3 - \bar{I}_3 = 8+2j$$

$$4\bar{I}_3 = 8+2j$$

$$\bar{I}_3 = \left(\frac{4+j}{2}\right)$$

$$\bar{I} = \bar{I}_2 - \bar{I}_3$$

$$= (2-j)\bar{I}_3 - \bar{I}_3$$

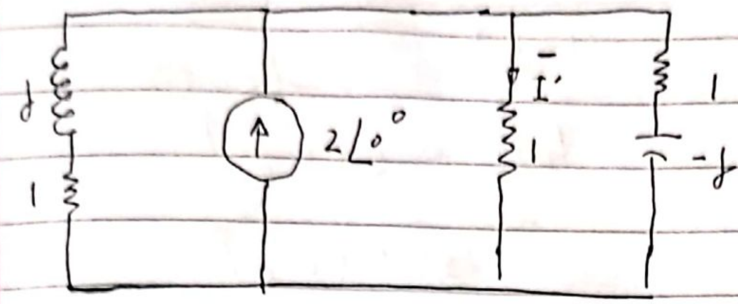
$$= (1-j)\bar{I}_3$$

$$= \frac{(1-j)(4+j)}{2}$$

$$= \frac{4+j-4j+1}{2}$$

$$\bar{I} = \frac{5-3j}{2}$$

Superposition

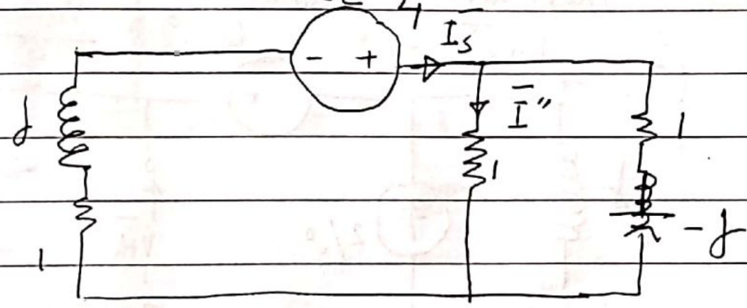


$$\bar{I}' = \left(\frac{\frac{1}{1}}{\frac{1}{1+j} + 1 + \frac{1}{1-j}} \right) 2\angle 0^\circ$$

$$= \frac{2\angle 0^\circ}{\frac{2}{2} + \frac{1-j}{2} + \frac{1+j}{2}}$$

$$= \frac{4\angle 0^\circ}{4} = 1\angle 0^\circ$$

6∠0°



$$I'' = \left(\frac{1-j}{2-j} \right) \bar{I}_s$$

$$\bar{I}_s = \left(\frac{2-j}{1-j} \right) I''$$

KVL :

$$\left(\frac{2-j}{1-j} \right)(1+j)\, I'' + I'' = 6\angle 0^\circ$$

$$\left[\frac{2+2j-j+1+1-j}{(1-j)}\right]\bar{I}'' = 6$$

$$4\bar{I}'' = 6(1-j)$$

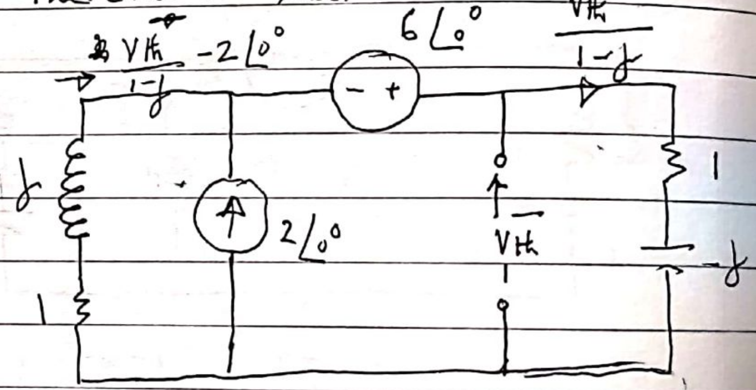
$$\bar{I}'' = \frac{3}{2}(1-j)$$

$$\bar{I} = 1+\frac{3}{2}(1-j)$$

$$= \frac{2+3-3j}{2}$$

$$\bar{I} = \frac{5-3j}{2}$$

Therenin's theorm :



$$(1+j)\left(\frac{\bar{V}_{th}}{1-j} - 2\right) + \bar{V}_{th} = 6\angle 0^\circ$$

$$\frac{(1+j)^2\,\bar{V}_{th}}{2} + \bar{V}_{th} = 6$$

$$+2(1+j)$$

$$\frac{(1+2j-1)\bar{V}_{th}}{2} + \bar{V}_{th} = 6+2+2j$$

$$(2+2j)\bar{V}_{th} = 12+4+4j$$

$$\bar{V}_{th} = \frac{16+4j}{2(1+j)}$$

$$= \frac{8+2j}{1+j}$$

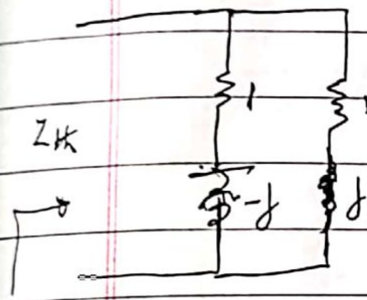
$$= \frac{(8+2j)}{1+j}\cdot\frac{(1-j)}{(1-j)}$$

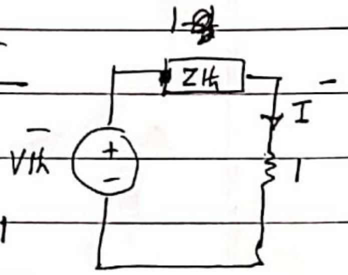
$$= \frac{8-8j+2j+2}{2}$$

$$= \frac{10-6j}{2}$$

$$\bar{V}_{th} = 5-3j$$

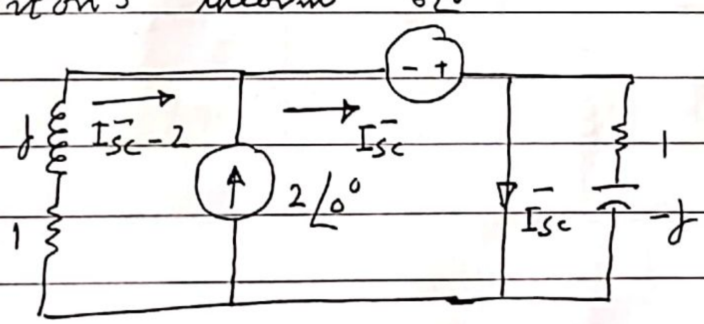
$$Z_{th} = \frac{(1-j)(1+j)}{2} = \frac{1+1}{2} = 1$$





$$\bar{I} = \frac{5-3j}{2}$$ ✓

Norton's theorm $6\angle 0^\circ$



KVL:

$$(1+j)(\bar{I}_{sc}-2) = 6\angle 0^\circ$$

$$\bar{I}_{sc} - 2 + j\bar{I}_{sc} - 2j = 6$$

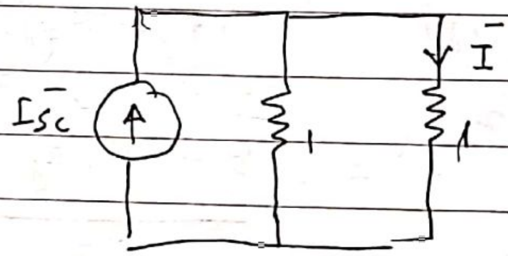
$$(1+j)\bar{I}_{sc} = 8 + 2j$$

$$\bar{I}_{sc} = \frac{8+2j}{1+j} = \frac{(8+2j)(1-j)}{2}$$

$$= \frac{8-8j+2j+2}{2}$$

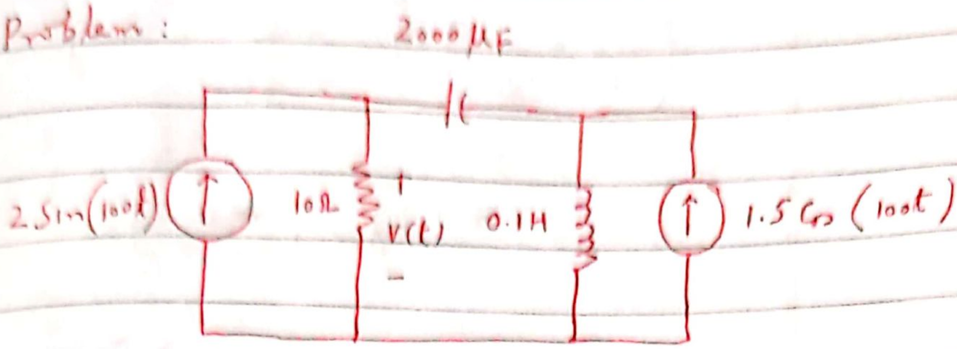
$$= \frac{10-6j}{2}$$

$$= 5-3j$$



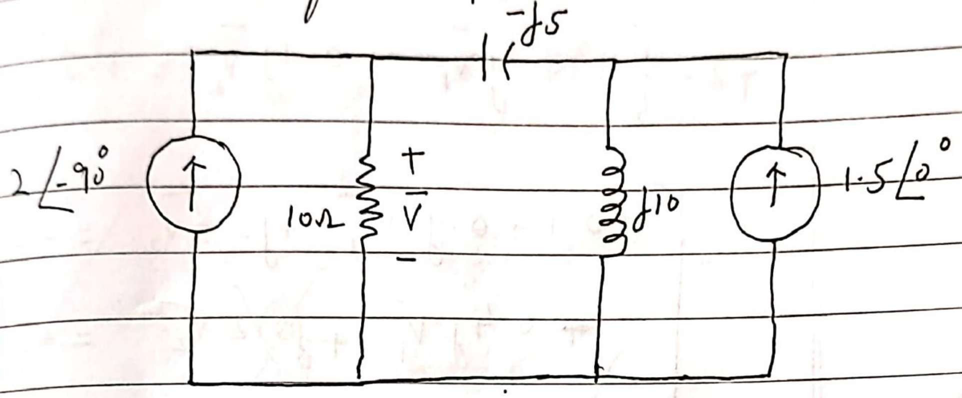
$$\bar{I} = \frac{5-3j}{2}$$

Problem :



Find V(t) using various methods.

Phasor equivalent circuit :

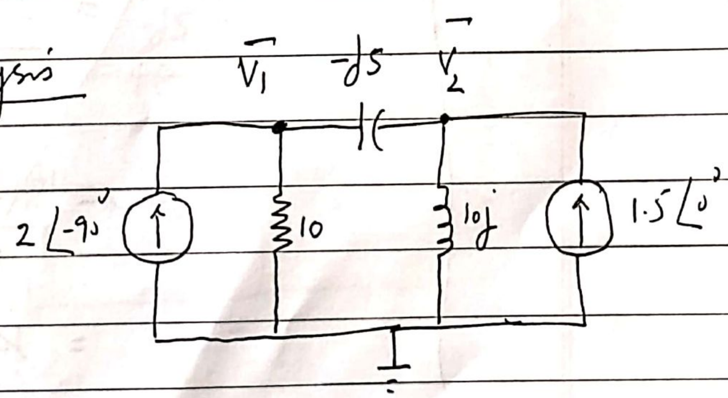


$$X_C = \frac{-j \times 10^6}{100 \times 2000} = -j5$$

$$X_L = j \times \cancel{2000} \times \cancel{10^{-6}} \times 100 \times 0.1 \ \Omega$$

$$= 10j \ \Omega$$

Nodal Analysis



$$\cancel{\frac{\bar{V}_1}{10} + \frac{\bar{V}_1 - \bar{V}_2}{-j5} =}$$

$$\left(\frac{1}{10}+\frac{1}{-j5}\right)\bar{V}_1 + \frac{\bar{V}_2}{j5} = -2j$$

$$(0.1+0.2j)\bar{V}_1 + 0.2j\bar{V}_2 = -2j \quad \text{— (1)}$$

$$\frac{\bar{V}_1}{j5} + \left(\frac{1}{10j} - \frac{1}{j5}\right)\bar{V}_2 = 1.5$$

$$-0.2j\bar{V}_1 + (0.2j - 0.1j)\bar{V}_2 = 1.5$$

$$+2\times\left[-0.2j\bar{V}_1 + 0.1j\bar{V}_2 = 1.5\right] \quad \text{— (2)}$$

$(0.1+j0.2)\bar{V_1} - j0.2\bar{V_2} = -j2$ — (1)

$2\left[-j0.2\bar{V_1} + j0.1\bar{V_2} = 1.5\right]$ — (2)

$-j0.4\bar{V_1} + j0.2\bar{V_2} = 3$

$(0.1-j0.2)\bar{V_1} = 3-j2$

$$\bar{V_1} = \frac{(3-j2)10}{1-j2}$$

$$= \frac{\cancel{10}^{2}(3-j2)(1+j2)}{\cancel{5}}$$

$= 2\left[3+j6-j2+4\right]$

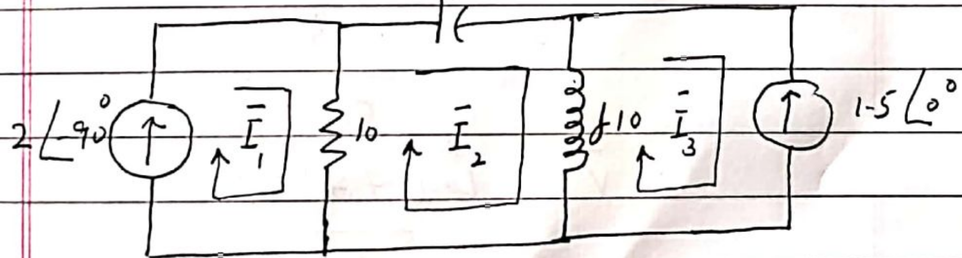
$= 14+j8$

$= 16.1 \angle 29.7^\circ$ check

$v(t) = 16.1\cos(100t+29.7^\circ)$

$16.1245\cos(100t+29.7449)$.

Mesh analysis:



$\bar{I_1} = 2\angle -90^\circ = -2j$ — (1)

$\bar{I_3} = -1.5\angle 0 = -1.5$ — (2)

$-10\bar{I_1} + (10+j5)\bar{I_2} - j10\bar{I_3} = 0$ (3)

from (3)

$$\cancel{j20 - j20 + 10 - j10\bar{I}_3 = 0}$$

$$j20 + (10 + j5)\bar{I}_2 + j15 = 0$$

$$(10 + j5)\bar{I}_2 = -j35$$

$$(2 + j)\bar{I}_2 = -j7$$

$$\bar{I}_2 = \frac{-j7}{2 + j}$$

$$\therefore \bar{I} = \bar{I}_1 - \bar{I}_2$$

$$= -2j + \frac{j7}{2+j}$$

$$= \frac{-4j + 2 + j7}{2 + j}$$

$$= (2 + j3)/(2 + j)$$

$$= \frac{(2 + j3)(2 - j)}{5}$$

$$= \frac{4 + j6 - 2j + 3}{5}$$

$$= \frac{7 + j4}{5}$$

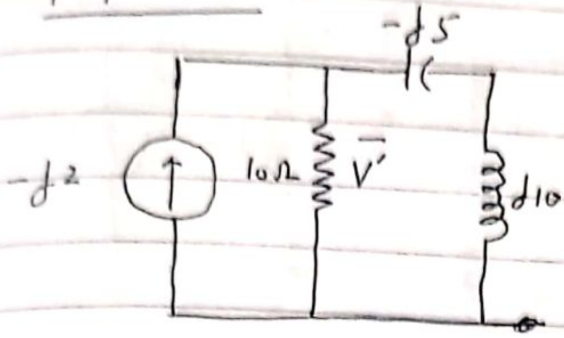
$$\therefore \bar{V} = \cancel{10}^{\,2}\frac{(7 + j4)}{\cancel{5}}$$

$$= 14 + j8$$

$$\bar{V} = 16.1 \angle 29.7^\circ$$

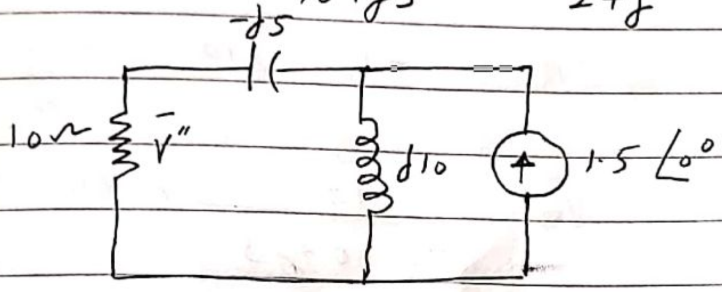
$$v(t) = 16.1 \cos(100t + 29.7^\circ)$$

Superposition



$$\bar{V}' = \underbrace{\left(\frac{j5}{10+j5}\right) \times -j2}_{\text{current divsn}} \times 10$$

$$= \frac{100}{10+j5} = \frac{20}{2+j}$$



$$\bar{V}'' = \underbrace{\left(\frac{j10}{10-j5+j10}\right) 1.5}_{\text{current divsn}} \times 10$$

$$= \frac{j150}{10+j5} = \frac{j30}{2+j}$$

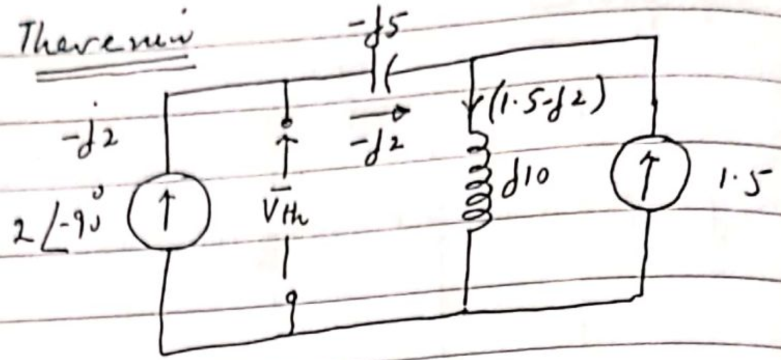
$$\bar{V} = \frac{20+j30}{2+j} = \frac{(20+j30)(2-j)}{5}$$

$$= \frac{40+j60-20j+30}{5}$$

$$= \frac{70+j40}{5} = 14+j8$$

$$= 16.1\angle 29.7^\circ$$

Thevenin



$$\bar{V}_{th} = (-j5)(-j2) + (1.5-j2)j10$$

$$= -10 + 15j + 20$$

$$= 10 + 15j$$

$$Z_{th} = -j5 + j10 = j5$$

$$\bar{V} = \frac{10}{10+j5} \times V_{th}$$

voltage division

$$= \left(\frac{10}{10+j5}\right) \times (10+j15)$$

$$= \frac{10\,(2+j3)}{(2+j)}$$

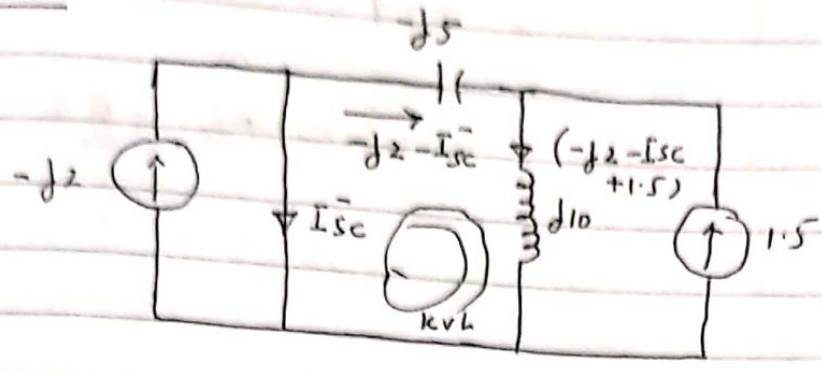
$$= \frac{\cancel{10}^{2}(2+j3)(2-j)}{\cancel{5}}$$

$$= 2\left[4 - 2j + 6j + 3\right]$$

$$= 2(7+4j)$$

$$= 14 + j8 = 16.1\angle 29.7^\circ$$

Norton



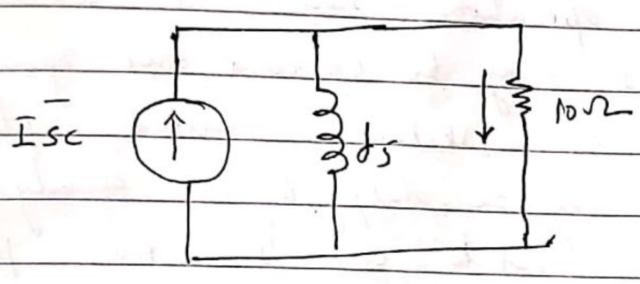
KVL

$$\left(-j2-\bar{I}_{sc}\right)\times \cancel{-j5}^{-1} + \left(-j2-\bar{I}_{sc}+1.5\right)\cancel{j10}^{2} = 0$$

$$j2+\bar{I}_{sc}-j4-2\bar{I}_{sc}+3=0$$

$$-\bar{I}_{sc}=-3+j2$$

$$\bar{I}_{sc}=3-j2$$



$$\bar{V}=\left(\frac{j5}{10+j5}\right)\bar{I}_{sc}\times 10$$

$$=\left(\frac{j5}{10+j5}\right)(3-j2)\times 10$$

$$=10\left(\frac{j}{2+j}\right)(3-j2)$$

$$=\frac{\cancel{10}^{2}(j)(2-j)(3-j2)}{\cancel{5}}$$

$$\bar{V} = 2[1 + j2][3 - j2]$$

$$= 2[3 - j2 + 6j + 4]$$

$$\cancel{= 2[10 + j}$$

$$= 2[7 + j4]$$

$$= 14 + j8 = 16.1\angle 29.7^\circ$$