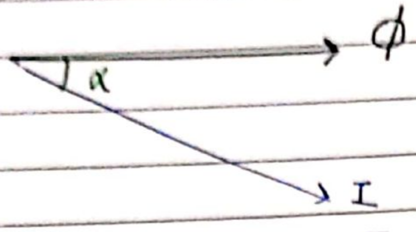
INDUCTION TYPE INSTRUMENTS



Amm. & voltmeter are suitable for AC only which depend on magnetic induction.

Most widely used application is energy meters.

Principle of Operation

$\phi = \phi_m \sin \omega t$ $\quad i = I_m \sin(\omega t - \alpha)$

Instantaneous torque $\propto$ product of current and flux.

$\therefore$ Inst. torque $\propto \phi i$

$\therefore$ " " $\propto \phi_m \cdot I_m \sin \omega t \cdot \sin(\omega t - \alpha)$

Mean torque over a cycle of period T

$$T_d \propto \frac{1}{T} \int_0^T \phi_m \cdot I_m \sin \omega t \sin(\omega t - \alpha)\, dt$$

$$\propto \frac{\phi_m \cdot I_m}{2} \cos \alpha.$$

$$T_d \propto \phi \cdot I \cos \alpha$$

(where $\phi = \frac{\phi_m}{\sqrt{2}}$, $I = \frac{I_m}{\sqrt{2}}$) rms values

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In all induction instruments, we have two fluxes produced by currents flowing in the windings of the instrument.

These fluxes are alternating in nature & so they produce emfs in a metallic disc or drum provided for this purpose.
These emfs in turn circulate circulate eddy currents in the disc / drum.

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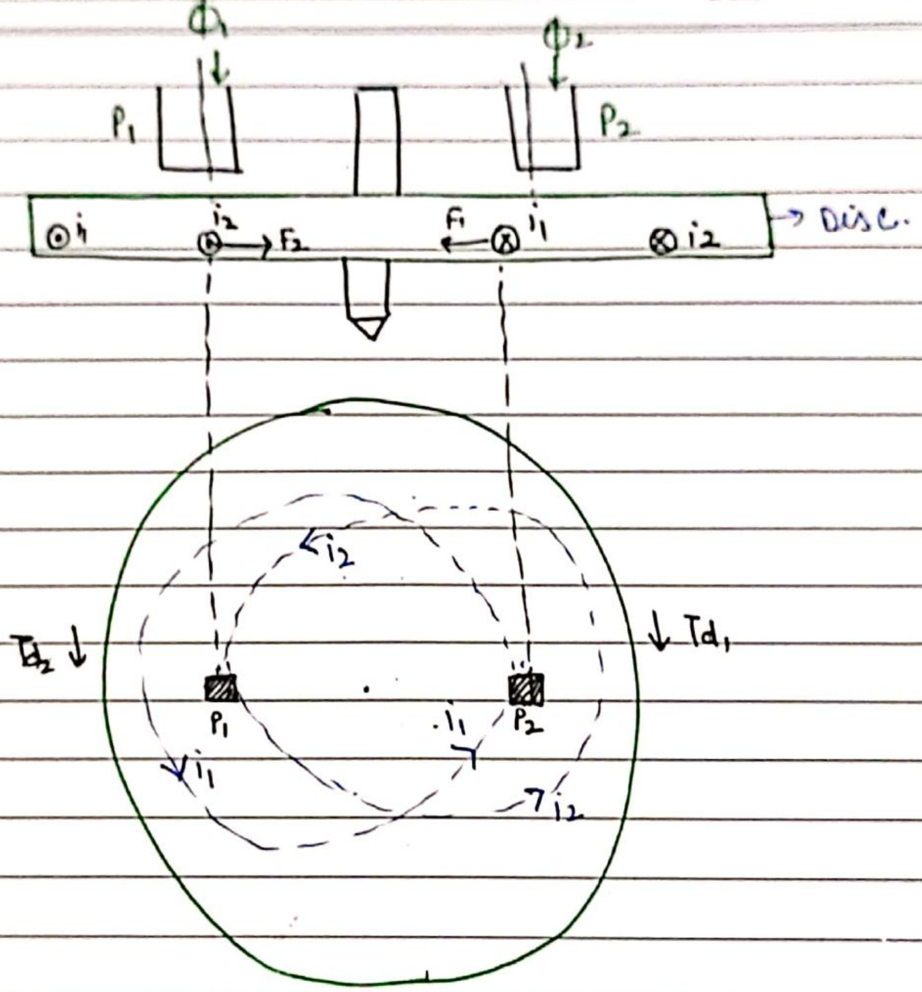

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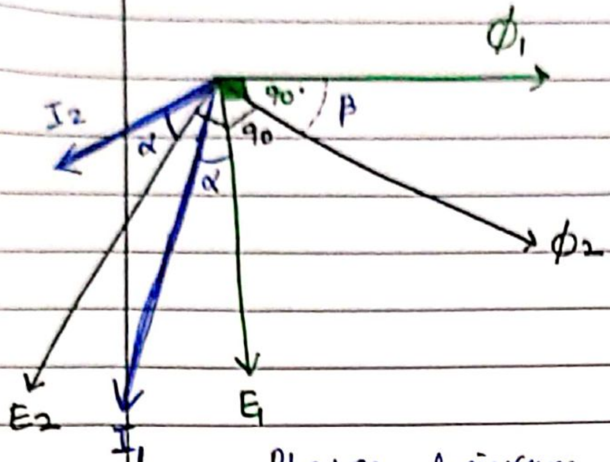
In the fig (1)



Phasor diagram

P_1 and P_2 are producing alternating fluxes ϕ_1 and ϕ_2 which cut the disc.

Any annular portion of the disc with the centre on the axis P_1 will be linked by alternating flux ϕ_1, hence it will have an induced emf & it will produce i_1

||ly ϕ_2 will produce i_2.

Thus we have two fluxes & two eddy currents & ∴ 2 torques are produced.

(1) ϕ_1 interacting with i_2.

(2) ϕ_2 " " i_1

∴ Total torque is the sum of these two torques.

$\phi_1 = \phi_{m_1} \sin \omega t$ $\qquad$ $\phi_2 = \phi_{m_2} \sin(\omega t - \beta)$

rms $\phi_1 = \frac{\phi_{m1}}{\sqrt{2}}$ $\qquad$ $\phi_2 = \frac{\phi_{m2}}{\sqrt{2}}$

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ϕ_1 produces emf in disc P by transformer action.

$e_1 \propto \left[-\frac{d\phi_1}{dt}\right] \propto \frac{-d}{dt}\left[\phi_{m_1} \sin wt\right]$

$\propto \left[-w\, \phi_{m_1} \cos wt\right] \propto \left[-f\, \phi_{m_1} \cos wt\right]$

It is clear than e_1 lags the flux by $90°$

E_1 = rms value of e_1

$\therefore E_1 \propto f \cdot \frac{\phi_{m_1}}{\sqrt{2}}$

$\therefore \boxed{E_1 \propto f \cdot \phi_1}$

eddy current i_1 has rms I_1.
Z = impedance of eddy current path.

$\therefore I_1 = \frac{E_1}{Z} \propto \frac{f}{Z}\phi_1$

I_1 lags E_1 by α.

The angle b/w ϕ_2 & I_1 is $(90-\beta+\alpha)$

$\therefore$ Average torque produced

$T_{d_1} = \phi_2 I_1 \cos(90-\beta+\alpha)$

$\therefore T_{d_1} \propto \phi_2 \cdot \phi_1 \frac{f}{Z} \cos(90-\beta+\alpha)$

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Now ϕ_2 produces emf E_2

$$E_2 \propto f \phi_2$$

E_2 lags ϕ_2 by 90.

$I_2 \propto \frac{f}{Z} \phi_2$.

I_2 lags E_2 by α.

$\therefore$ Angle b/w ϕ_1 and I_2 is $(90 + \beta + \alpha)$

$\therefore$ Average torque produced by ϕ_2 & I_2

$T_{d_2} \propto \phi_1 I_2 \cos(90 + \beta + \alpha)$

$\propto \phi_1 \phi_2 \frac{f}{Z} \cos(90 + \alpha + \beta)$

Let fluxes ϕ_1, ϕ_2 be increasing in direction shown.

ϕ_2 & I_1 → forces from Right to left

ϕ_1 & I_2 → " " left to Ri

Thus total torque is the difference of two torques.

$T_d = T_{d_1} - T_{d_2}$

$T_d \propto \phi_1 \phi_2 \cdot \frac{f}{Z} \cos(90 - \beta + \alpha) - \phi_2 \phi_1 \frac{f}{Z}$

$\cos(90 + \beta$

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$$T_d \propto \phi_2 \phi_1 \frac{f}{z} \left[\sin(\beta + \alpha) + \sin(\beta + \alpha) \right]$$

$$\propto \phi_1 \phi_2 \frac{f}{z} \sin\beta \cos\alpha$$

OBSERVATIONS

(a) $\therefore T_d \propto \cos\alpha$

$\therefore$ To $\uparrow T_d$; α should be ≈ 0.

$\therefore$ Path of eddy currents must be highly resistive.

(b) Torque $\propto \sin\beta$.

$\therefore T_d \uparrow \Rightarrow \beta \approx 90^\circ$

β = angle b/w ϕ_1 & ϕ_2.

$\therefore$ Two fluxes must be displaced by 90°.

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ELECTROSTATIC INSTRUMENTS

The deflecting torque is produced by the action of electric field on charged conductors.
Such instruments are essentially voltmeter.

This is the only instrument that measures voltage directly rather than by the effect of the current it produces.

i) It consumes no power (except during brief transient period)

ii) It represents infinite impedance to the circuit under measurement.

iii) Its action depends on the reaction between two electrically charged bodies.

Electrostatic mechanism resembles a variable capacitor, where the force existing b/w two ∥ plates is a function of the potential difference applied to them.

FORCE AND TORQUE EQUATIONS

(A) Linear Motion

Let a potential difference of V volts is applied between A & B. Then a force ~~applied~~ of attraction F Newtons exists between them.

Plate B moves towards plate A until this force is balanced by that of spring.

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Let capacitance of the capacitor, then energy stored $= \frac{1}{2} CV^2$ Joules.

Let there by a small increment dv in the applied voltage. Then plate B will move a small distance dx towards A.

A capacitive current flows, given by

$$i = \frac{dq}{dt} = \frac{d}{dt}(CV) = C\frac{dV}{dt} + V\frac{dC}{dt}.$$

Input energy $= Vi\,dt = V(C\,dV + V\,dC)$

$= V^2 dC + CV\,dV$

Change in stored energy $= \frac{1}{2}(C+dC)(V+dV)^2 - \frac{1}{2}CV^2$

$= \frac{1}{2}(C+dC)(V^2+2VdV+dV^2) - \frac{1}{2}CV^2$

Neglecting higher order terms.

$= \frac{1}{2}(CV^2 + 2VC\,dV + C\,dV^2 + V^2 dC + 2V\,dV.dC + dC\,dV^2) - \frac{1}{2}CV^2$

$= \frac{1}{2}V^2 dC + CV.dV$

From principle of conservation of energy.

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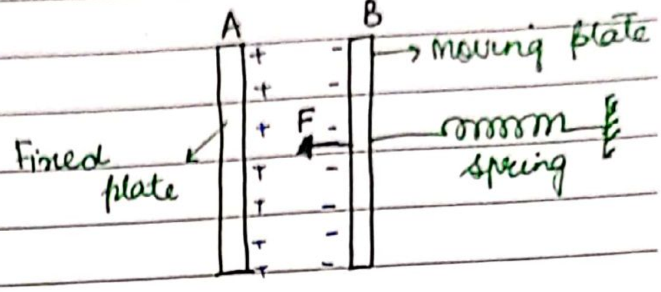
Input energy = change in stored energy + mechanical work done.

$$V^2 dC + CV dV = \frac{1}{2}V^2 dC + CV dV + F dx$$

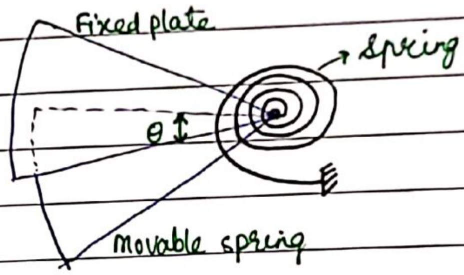
$$F \cdot dx = \frac{1}{2}V^2 dC$$

$$\boxed{F = \frac{1}{2}V^2 \cdot \frac{dC}{dx}}$$



(A) ROTATIONAL MOTION

The above treatment can be applied to the rotational motion by writing an angular displacement θ in place of linear displacement 'x' and deflecting torque Td in place of force F.



$$T_d = \frac{1}{2}V^2 \cdot \frac{dC}{d\theta} \quad N \cdot m$$

$$\theta \Rightarrow T_c = K\theta \quad \text{(spring control)}$$

$$\therefore K\theta = \frac{1}{2}V^2 \cdot \frac{dC}{d\theta}$$

$$\boxed{\theta = \frac{1}{2}\left(\frac{V^2}{K}\right) \cdot \frac{dC}{d\theta}}$$

- Use of the instrument is limited to certain special applications particularly in ac circuits of relatively high voltage where the current taken by other instrument would result in erroneous indications. A ~~particular resis~~

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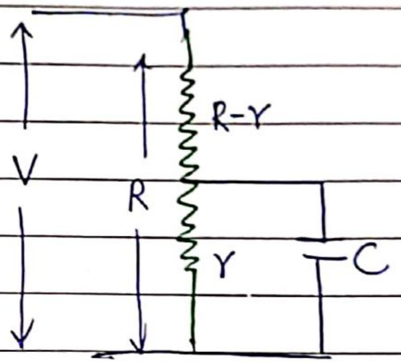
A protective resistor is used in series with the instrument to limit the current in case of a short circuit b/w the plates.

EXTENSION OF THE RANGE OF ELECTROSTATIC VOLTMETERS.

The voltage range of Elec. Voltmeter may be extended by :-

i) Resistance Potential Divider

V = Voltage to be measured.
R = total resistance.



Electrostatic voltmeter of ~~resistance~~ & capacitance 'C' is connected across a portion of the potential divider having resistance 'r'.

For operation on dc :- assuming electrostatic voltmeter of capacitance 'C' has infinite leakage resistance, Voltage multiplying factor of m is equal to $\left(\frac{R}{r}\right)$.

Operation on ac : There is shunting effect across the resistance by the capacitive reactance of the voltmeter.

$$Z = \frac{r \cdot \frac{1}{j\omega C}}{r + \frac{1}{j\omega C}} = \frac{r}{1 + j\omega C r}$$

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Total impedance Z_i connected across voltage V is , $Z_t = (R-r) + \frac{r}{1+j\omega Cr}$

$$= \frac{R + j\omega Cr\,(R-r)}{1+j\omega Cr}$$

Thus ; $\frac{Z_t}{Z} = \frac{R + j\omega Cr\,(R-r)}{r}$

$$= \left(\frac{R}{r}\right) + j\left[\omega C\,(R-r)\right]$$

Magnitude of $\left(\frac{Z_t}{Z}\right) = \sqrt{\left(\frac{R}{r}\right)^2 + \omega^2 C^2 (R-r)^2}$

Voltage multiplying factor 'm' is given by

$$m = \left|\frac{Z_t}{Z}\right| = \sqrt{(R/r)^2 + \omega^2 C^2 (R-r)^2}$$

$$= \frac{R}{r}\sqrt{1 + \frac{\omega^2 C^2 r^2}{R^2}(R-r)^2}$$

As electrostatic voltmeter is normally connected across a small portion of the voltage divider.

$$\left(\frac{R-r}{R}\right) \simeq 1$$

When $\omega^2 C^2 r^2 \leq 1$ or $r \leq \frac{1}{\omega C}$

$$\therefore m = \left(\frac{R}{r}\right)$$

$r < \left(\frac{1}{\omega C}\right)$ means that elec. voltmeter will not

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offer appreciable loading to the portion of potential divider across which it is connected

When 'r' is low, ∴ total R of potential divider will be low for a given value of m.

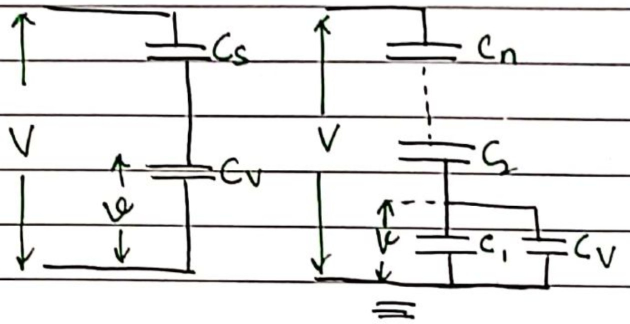
∴ For a given voltage V, $P = \left(\frac{V^2}{R}\right)$ would be high

∴ voltage range of resistance pot divider is limited to few kV.

2. Capacitive Multipliers.

C_v = capacitance of voltmeter

C_s : capacitance of series capacitance



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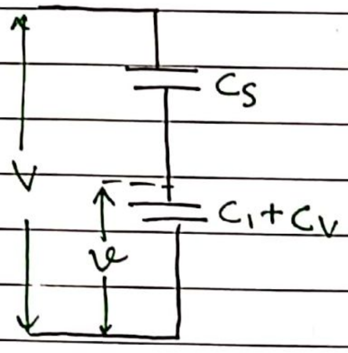

Fig (b)

Z = impedance of voltmeter

$$Z = \frac{1}{j\omega C_v}$$

Total capacitance

$$C_t = \frac{C_s \cdot C_v}{C_s + C_v}.$$

$$\therefore Z_t = \text{total impedance} = \frac{C_s + C_v}{j\omega C_s C_v}$$

$$\therefore \left(\frac{Z_t}{Z}\right) = \frac{C_s + C_v}{j\omega C_s \cdot C_v} \bigg/ \frac{1}{j\omega C_v} = \frac{C_s + C_v}{C_s}$$

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$\therefore$ Multiplying factor $m = \frac{V}{v} = \frac{Z_t}{Z} = \frac{C_s + C_v}{C_s}$

$= 1 + \frac{C_v}{C_s}$.

C_v varies with deflection of the voltmeter.
$\therefore$ Elec. voltmeter should be calibrated together with its series multiplier capacitor

· In practice, capacitor multiplier consists a set of capacitors connected in series across the voltage to be measured & elec. voltmeter is connected across one of them as shown in fig (b).

Cap. b/w C_1 and $C_v = C_1 + C_v$
$\therefore$ equivalent C_s with this // combination

$$\frac{1}{C_s} = \frac{1}{C_2} + \frac{1}{C_3} + \cdots \frac{1}{C_n}$$

multiplying factor 'm' is obtained by replacing C_v by $C_1 + C_v$

$$m = 1 + \frac{C_v'}{C_s}$$

$$= 1 + \frac{C_1 + C_v}{C_s}$$

C_v varies deflection $\therefore$ voltmeter should be calibrated with its multiplier.

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Errors: electrostatic instruments are free from errors. Effects due to temperature, frequency, stray mag fields are absent.
But screening has to be provided for stray electrostatic fields.

Advantages

1. They draw negligible power from mains. The initial current drawn by them when connected across d.c. mains is the charging current which dies down gradually depending upon insulation resistance of meter ($\approx 10^{12}\,\Omega$). The currents drawn on a.c. are small and depend upon impedance of meter circuit.
2. Used on both a.c and d.c.
3. No frequency error and wave-form error as the deflection is proportional to square of voltage and there is no hysteresis.
4. No errors caused by stray magnetic fields as the instrument works on electrostatic principle.
5. Used particularly for higher voltages.

Disadvantages:

1. Use restricted to certain special applications particularly in a.c. circuits of high voltage where current drawn by ~~them is~~ other instruments would result in erroneous indications.
2. These instruments are expensive, large in size and are not robust in construction.
3. Scale is not uniform.
4. The operating forces are small as compared to operating forces in electromagnetic type instruments.

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