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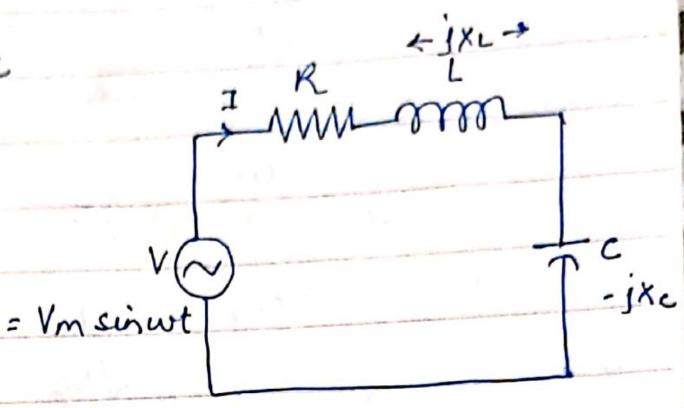
RESONANCE IN A.C CIRCUITS :-

i) Series Resonance



$$\bar{I} = \frac{\bar{V}}{R + j(X_L - X_c)}$$

$$\bar{I} = \frac{\bar{V}}{R + j\left(\omega L - \frac{1}{\omega C}\right)} = \frac{\bar{V}}{\bar{Z}}$$

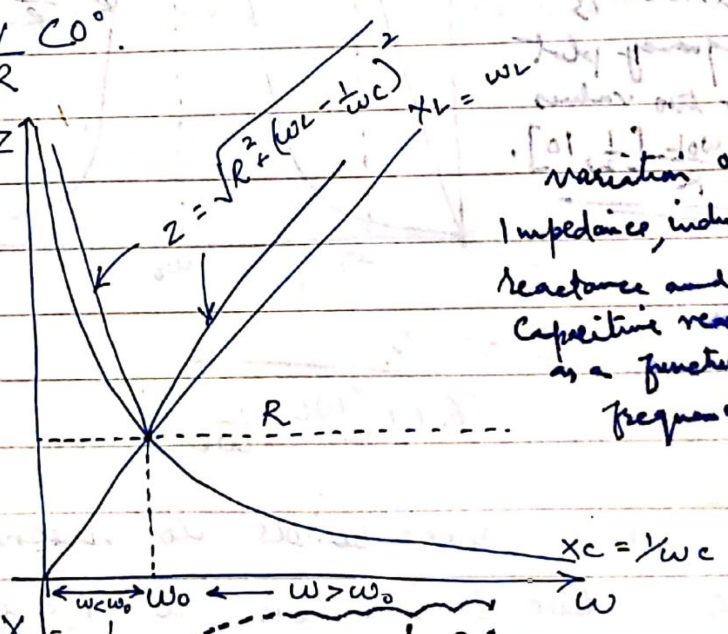
if $\omega L - \frac{1}{\omega C} = 0$ $\quad \{X_L - X_C = 0\}$

$$\bar{I} = \frac{\bar{V}}{R} = \frac{V\angle 0^\circ}{R\angle 0^\circ}$$

$$\bar{I} = \frac{V}{R}\angle 0^\circ.$$

Power factor = 1

ie, $\cos 0^\circ = 1$



variation of Impedance, inductive reactance and capacitive reactance as a function of frequency;

$X_L = \omega_L$, $X_C = \frac{1}{\omega C}$ $\quad -X_C = -\frac{1}{\omega C}$

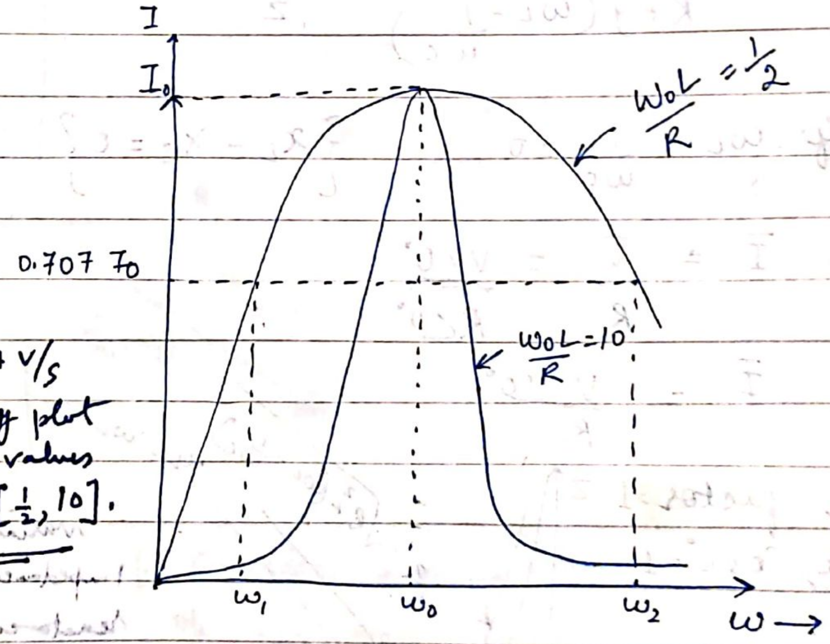
$$\omega L - \frac{1}{\omega C} = 0$$

$$\omega^2 = \frac{1}{LC}$$

$$\omega = \frac{1}{\sqrt{LC}} \quad (\text{say } \omega_0)$$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$\boxed{f_0 = \frac{1}{2\pi\sqrt{LC}}}$$



Current v/s frequency plot for two values of $\frac{\omega_0 L}{R}$ $[\frac{1}{2}, 10]$.

$$I = \frac{V}{R + j\left(\omega L - \frac{1}{\omega C}\right)}$$

ω_0 corresponds to resonance frequency

At two frequencies ω_1 ($< \omega_0$) and ω_2 ($> \omega_0$)

the currents are same.

Band Width :- For a series RLC circuit band width is defined as the range of frequency for $\leq$ the power delivered to $R \geq P_0/2$ where P_0 is the power delivered to R at resonance.

Let currents I_1 and I_2 correspond to the frequencies ω_1 and ω_2

then, $I_1^2 R = I_2^2 R = \frac{1}{2} I_0^2 R$

$$I_1 = I_2 = \frac{I_0}{\sqrt{2}} = 0.707\, I_0$$

Thus, band width may be identified on the resonance curve as that range of frequency over $\leq$ the mag. of current ≥ 0.707 times the current at resonance.

$$B.W = \omega_2 - \omega_1$$

$$I\omega_1 = 0.707\, I_0 = \frac{V}{\sqrt{R^2 + \left(\omega_1 L - \frac{1}{\omega_1 C}\right)^2}}$$

or $$\frac{V}{\sqrt{R^2 + \left(\omega_1 L - \frac{1}{\omega_1 C}\right)^2}} = 0.707 \frac{V_0}{R} \qquad \left(\because I_0 = \frac{V_0}{R} = V/R\right)$$

$$= 0.707\, V/R$$

$$\Rightarrow \frac{1}{\sqrt{R^2+\left(\omega_1 L - \frac{1}{\omega_1 C}\right)^2}} = \frac{0.707}{R}$$

$$\frac{R^2}{(0.707)^2} = R^2 + \left(\omega_1 L - \frac{1}{\omega_1 C}\right)^2$$

$$\left(\omega_1 L - \frac{1}{\omega_1 C}\right)^2 = R^2$$

$$\omega_1 L - \frac{1}{\omega_1 C} = \pm R$$

Now,

$$\omega_1 L - \frac{1}{\omega_1 C} = -R$$ capactive dominant

$$\frac{\omega_1^2 LC - 1}{\omega_1 C} = -R$$

$$\omega_1^2 LC - 1 + CR\omega_1 = 0$$

$$\omega_1^2 LC + \omega_1 CR - 1 = 0.$$

$$\omega_1 = \frac{-CR \pm \sqrt{C^2R^2 + 4LC}}{2LC}$$

$$\omega_1 = -\frac{R}{2L} \pm \sqrt{\frac{1}{LC} + \left(\frac{R}{2L}\right)^2}$$

$$\Rightarrow \omega_1 = -R/2L + \sqrt{1/LC + (R/2L)^2}$$ } freq. can't be -ve

Also, $$\omega_2 L - \frac{1}{\omega_2 C} = +R$$ (inductive dominant)

$$\omega_2^2 LC - \omega_2 RC - 1 = 0$$

$$\omega_2 = \frac{RC \pm \sqrt{R^2C^2 + 4LC}}{\cancel{2R}2LC}$$

$$\omega_1 = \frac{R}{2L} \pm \sqrt{\frac{1}{LC} + \left(\frac{R}{2L}\right)^2}$$

$$\omega_2 = \frac{R}{2L} + \sqrt{\frac{1}{LC} + \left(\frac{R}{2L}\right)^2}$$

} freq can't be -ve.

Now, $BW = W_2 - \omega_1$

$$BW = \frac{R}{2L} + \sqrt{\frac{1}{LC} + \left(\frac{R}{2L}\right)^2} - \left(-\frac{R}{2L} + \sqrt{\frac{1}{LC} + \left(\frac{R}{2L}\right)^2}\right)$$

$$B.W = \frac{R}{L}$$

Quality Factor (Q)

It is defined as the ratio of resonant frequency to band width and is a measure of selectivity or sharpness of tuning of the series RLC circuit.

$$Q_0 = \frac{W_0}{B.W} \qquad \frac{\omega_0}{1/RC} = \omega_0 RC$$

$$= \frac{W_0}{R/L}$$

$$= \frac{W_0 L}{R}$$

It finds application in communication and radio circuits.

Voltage drops across L and C at resonance :-

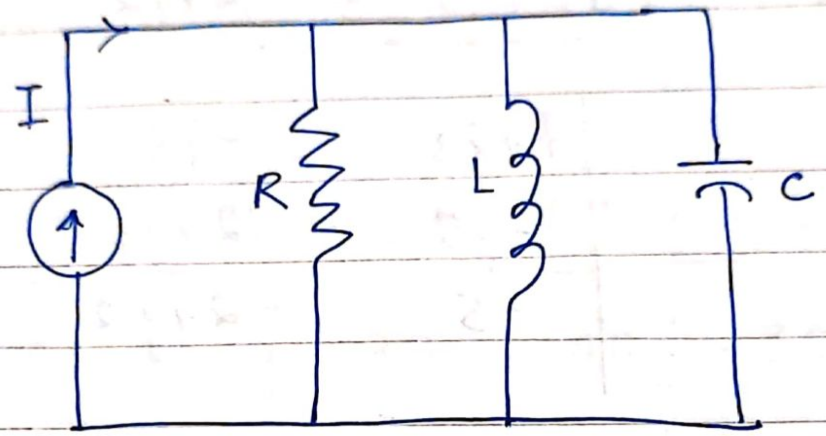
$$V_{L0} = I_0 \times (jX_L)$$
$$= \frac{V}{R} \times \omega_0 L \angle 90^\circ$$
$$= \frac{\omega_0 L}{R} \cdot V \angle 90^\circ$$
$$= Q_0 V \angle 90^\circ$$

$$V_{C0} = I_0 \times (-jX_C)$$
$$= \frac{V}{R} \times \frac{1}{\omega_0 C} \angle -90^\circ$$
$$= \frac{V}{R} (\omega_0 L) \angle -90^\circ \quad \text{(under resonance)}$$
$$= Q_0 V \angle -90^\circ.$$

Thus, voltage is amplified by a factor Q_0 at resonance.

$$Q_0 = \frac{V_{L0}}{V} ; \quad Q_0 = \frac{V_{C0}}{V} ;$$

ii) Parallel Resonance



(Do yourself)

5th Nov'03

Power in Sinusoidal Steady State circuits :-

Let $v(t) = V_m \cos\omega t$.

$i(t) = I_m \cos(\omega t - \theta)$

$$P(t) = v(t)\, i(t)$$

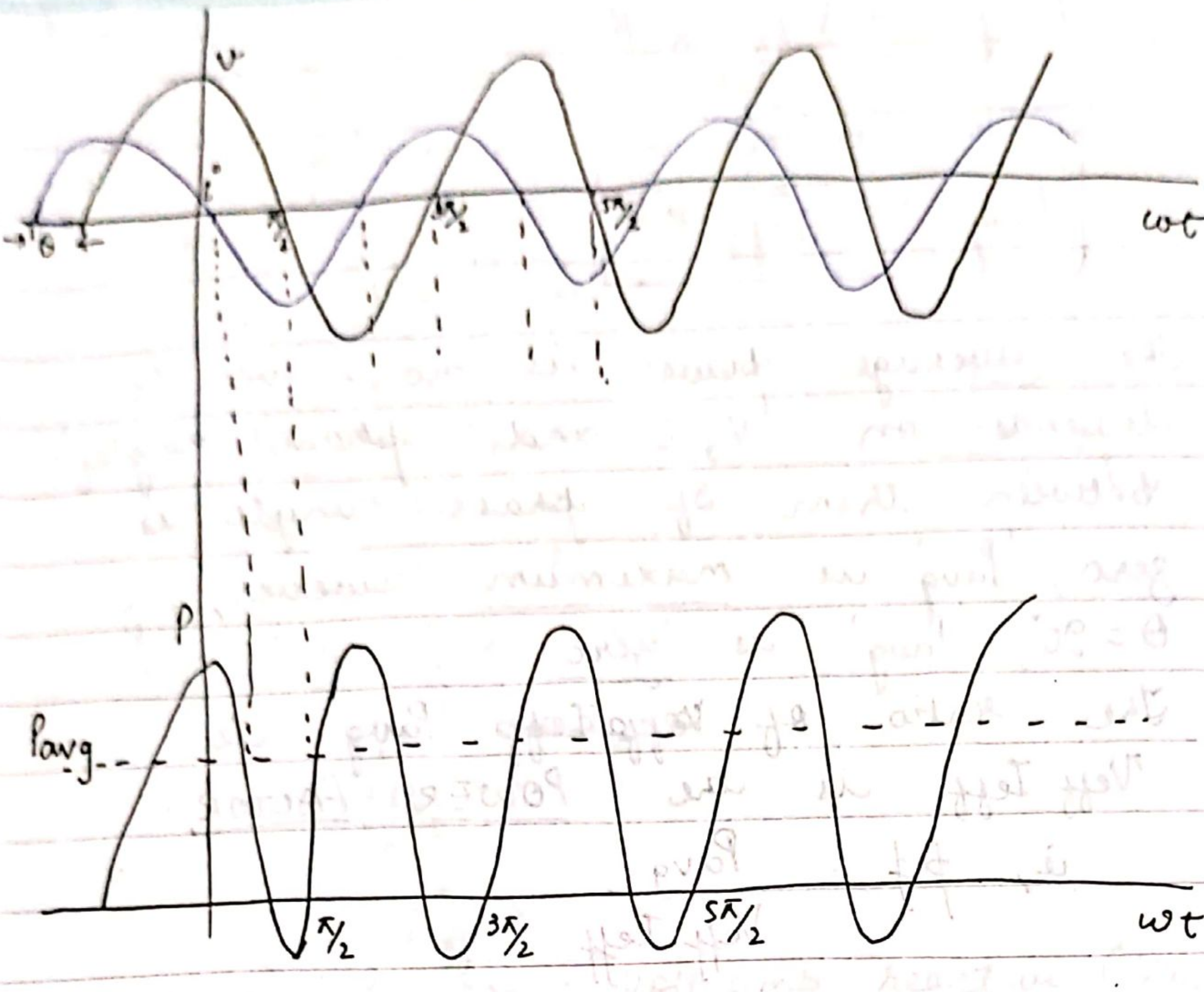
$$= V_m I_m \cos\omega t \cos(\omega t - \theta)$$

$$= \frac{V_m I_m}{2}\left[\cos\theta + \cos(2\omega t - \theta)\right]$$

$$P(t) = V_{eff} I_{eff} \cos\theta + V_{eff} I_{eff} \cos(2\omega t - \theta)$$

During a portion of one cycle, the instantaneous power is +ve $\Rightarrow$ means that power flows into the load. During the rest of the cycle the instantaneous power may be -ve $\Rightarrow$ indicates that the power flows out of the load. The net flow of power during one cycle is however non--ve and is called the average power

$$\boxed{P_{avg} = V_{eff} I_{eff} \cos\theta}$$



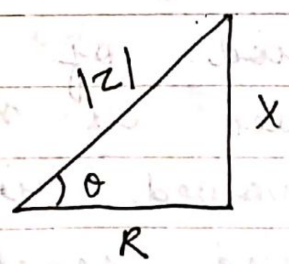
AVERAGE OR REAL POWER:-

The net or average power entering a load during one period is called the real power

$$P_{avg} = V_{eff} \, I_{eff} \cos\theta$$

If $z = R + jX = |z| \angle \theta$.

$$\cos\theta = \frac{R}{|z|}$$



$$P_{avg} = V_{eff} \times \frac{V_{eff}}{|z|} \times \frac{R}{|z|}$$

$$P_{avg} = \frac{V_{eff}^2}{|Z|^2} \times R$$

$$\boxed{P_{avg} = I_{eff}^2 \times R}$$

The average power is non -ve. It depends on V, I and phase angle between them. If phase angle is zero, 'P_{avg}' is <u>maximum</u>. However, if $\theta = 90°$, 'P_{avg}' is <u>'zero'</u>.

The ratio of ~~V_{eff} I_{eff}~~ P_{avg} to $V_{eff} I_{eff}$ is the <u>POWER</u> <u>FACTOR</u>.

ie, $p.f = \frac{P_{avg}}{V_{eff} I_{eff}}$

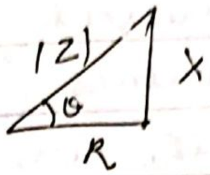
$$= 0 \leq p.f \leq 1.$$

<u>Reactive</u> <u>Power</u> :-

If a passive n/w contains inductors, capacitors or both, a portion of energy entering it during one cycle is stored and then returned to the source. During the period of energy return, the power is negative. The power involved in this exchange is called reactive or quadrature power.

Although the net effect of reactive power is zero, it degrades the operation of power systems. Reactive power is denoted by Q & defined as;

$$Q = V_{eff} \times I_{eff} \times \sin\theta.$$



If $Z = R + jX = |Z|\angle\theta$;

then, $\sin\theta = \dfrac{X}{|Z|}$

$$Q = V_{eff} \times \frac{V_{eff}}{|Z|} \cdot \frac{X}{|Z|}$$

$$= \frac{V_{eff}^2}{|Z|^2} \cdot X$$

$$= I_{eff}^2 X.$$

units of Q are = Volt amp reactive (Var)

Reactive power Q depends upon V, I and phase angle b/w them. It is the product of voltage & that component of current which is 90° out of phase with voltage thats why we call it quadrature power. Q is zero for $\theta = 0$. This occurs for a pure resistive load and when $\bar{V}$ and $\bar{I}$ are in phase. When the load is purely reactive, $\theta = 90°$ and Q attains its maximum magnitude for given V & I.

~~If current lags the voltage~~

Note that while P is always non -ve, Q can assume positive values (for an inductive load) where the current lags the voltage) or -ve values (for a capacitive load where the current leads the voltage). It is also customary to specify Q by its magnitude and load type.

Thus, if;

$Q = 100$ KVAR inductive

$= +100$ KVAR (lagging Vars)

or

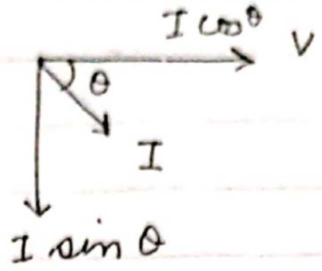
$Q = 50$ KVAR capactive

$= -50$ KVAR (leading Vars).

COMPLEX POWER; APPARENT POWER & POWER TRIANGLE:-

The two components P & Q of power play different roles & may not be added together. However they may be conveniently be brought together in the form of a vector quantity called complex power 'S' & is defined as;

$$S = P + jQ$$

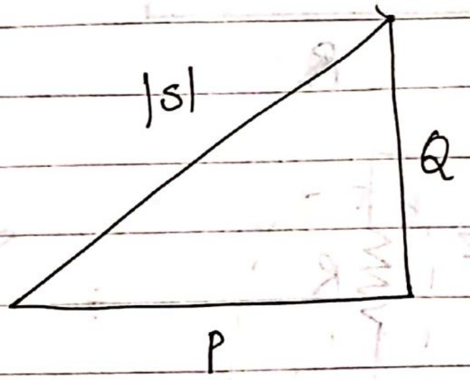


$$|S| = \sqrt{P^2 + Q^2}$$

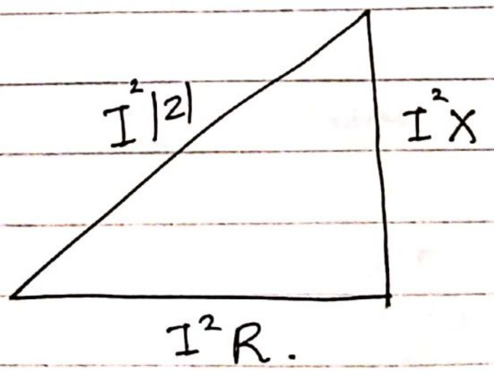
$$= V_{eff} I_{eff}$$

$$= \text{apparent power.}$$

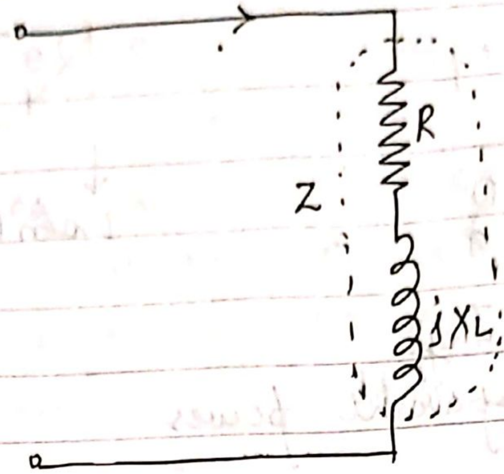
These three quantities (scalar) |S|, P and Q may be represented geometrically as the hypotenuse, base and perpendicular respectively of a right triangle, c we call as power triangle.

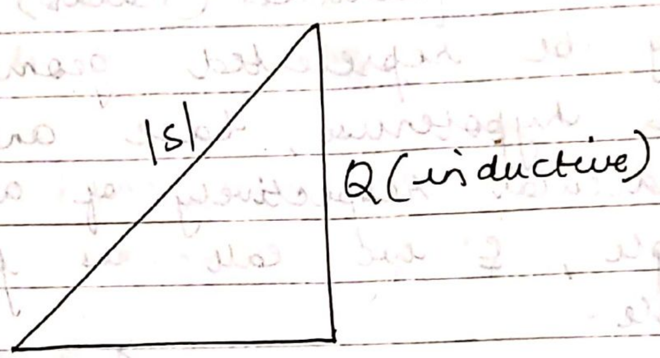


$$\Downarrow$$

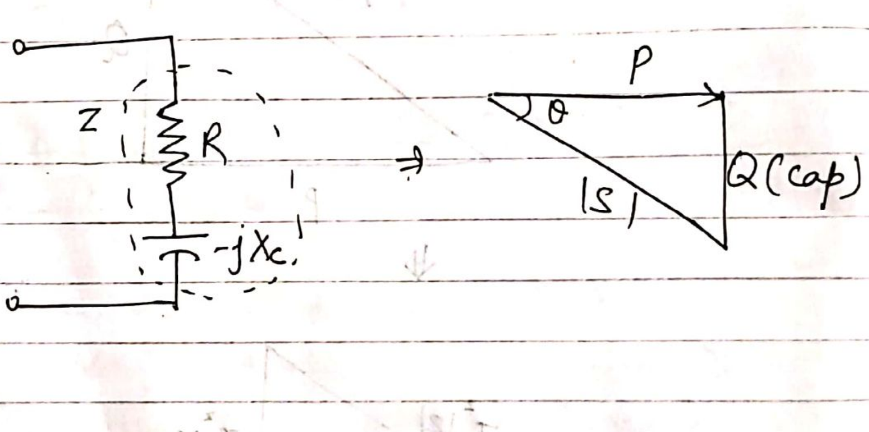


Ex 1



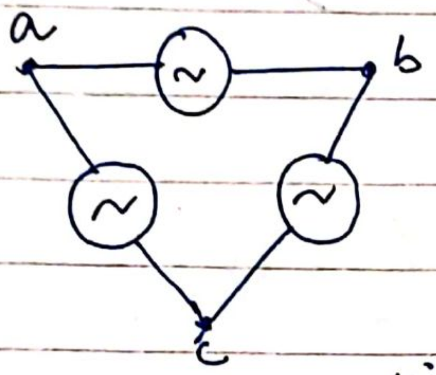


Ex2

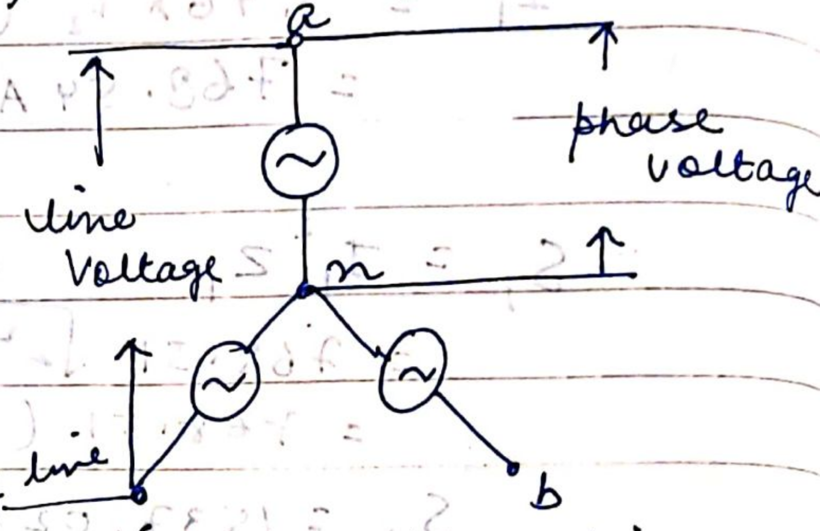


12th Nov'03

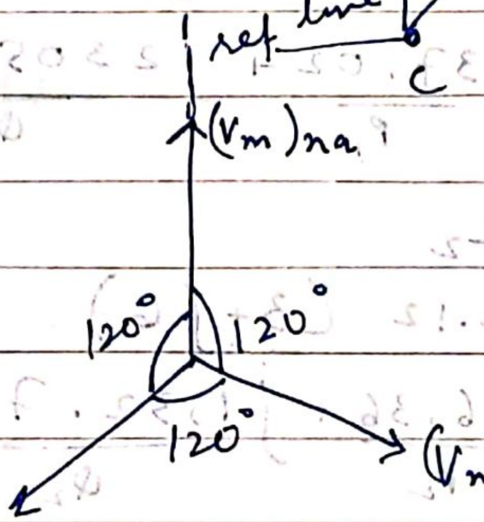
Balanced 3-phase circuits :-

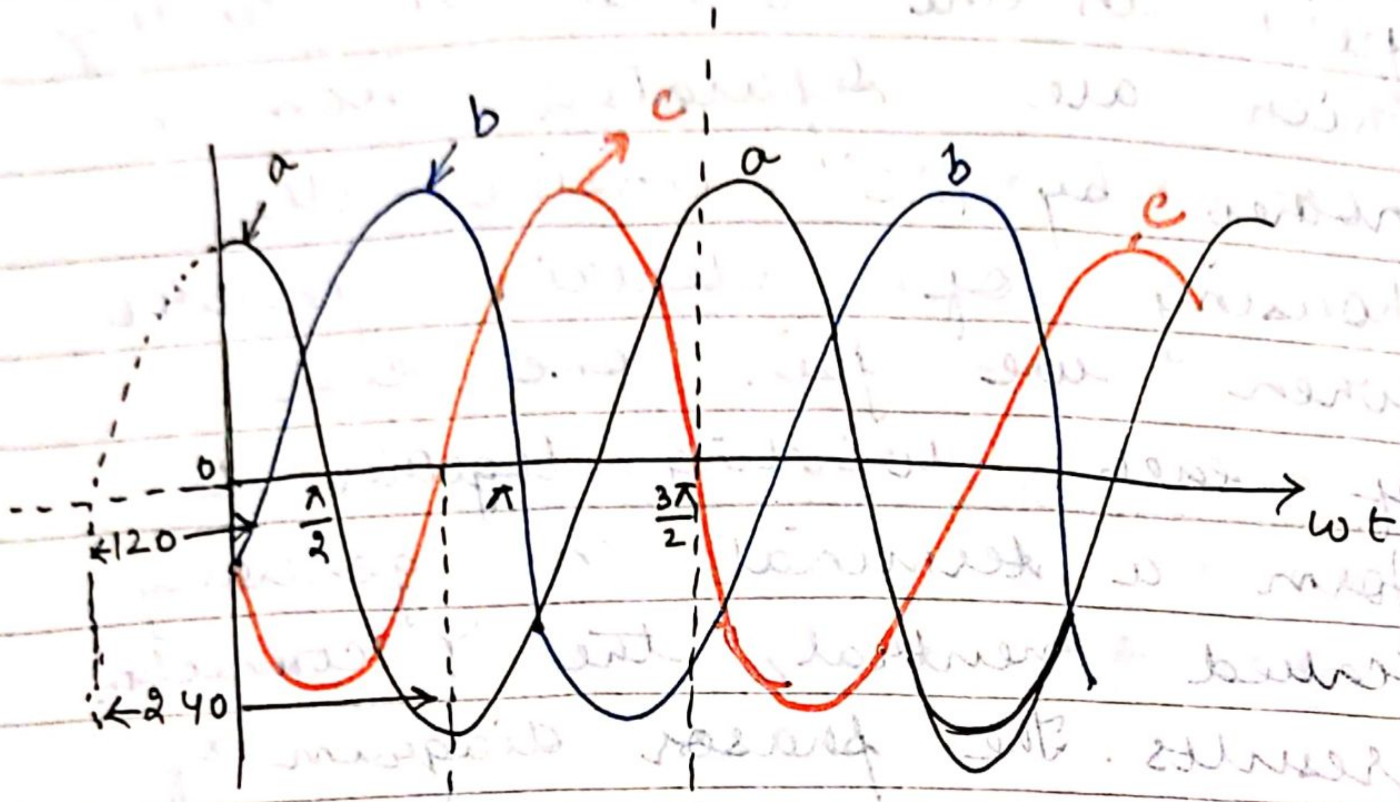


Δ - connection



Y - connection





phase sequence - abc

or

phase order

or

phase rotation.

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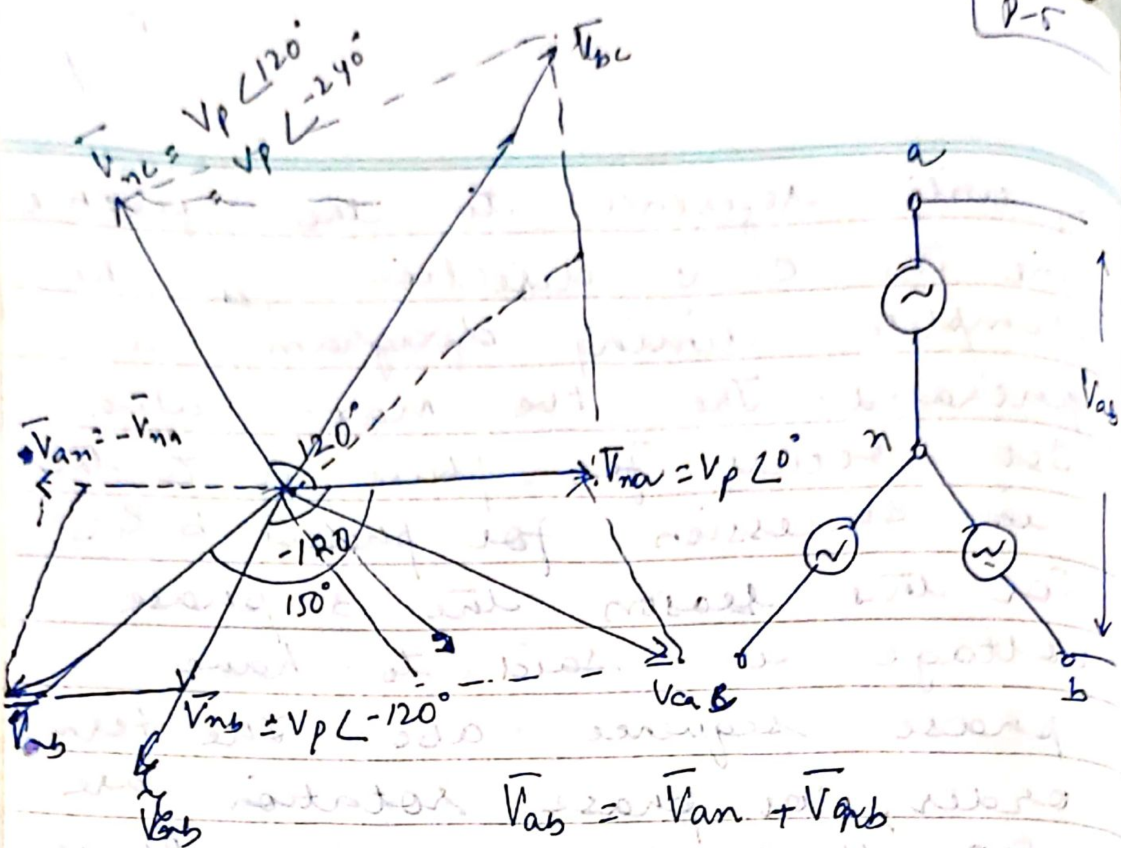
A balanced voltage 3-phase system is composed of 3 single phase voltages having same amplitude & frequency but time displaced from one another by 120°. These single phase voltages are generated by a common rotating flux field in the 3 identical windings which are separated from one another by 120°. Inside the housing of electric generator when we join one end of each winding together to form a terminal n generally called neutral, the Y connection results. The phasor diagram of this set of voltages and its subsequent timing diagram are also shown. For the time instant when phase a is in phase with vertical ~~phase~~ reference line, phase a is at its max. value while simultaneously phases b & c have values of (-) half $(-\frac{1}{2})$ the max. value. As the phasors revolve at angular frequency w, with reference to the reference line in CCW direction & the complete timing diagram is generated. The +ve max. value 1st occurs for phase a & then in succession for phases b & c. For this reason the 3 phase voltage is said to have phase sequence abc. The terms order, or phase rotation are also used synonimously. Phase sequence is imp. in certain applications for eg in 3 phase induction motors it determines whether the motor ~~forces~~ turns in CW or CCW

Line Voltage:- V_{ab}, V_{ac}, V_{cb} are the line voltages

__Current and Voltage Relationship in a Y connected 3-phase system:-__

V_{na} → voltage rise from n to a

$-\bar{V}_{na} = \bar{V}_{an}$ = voltage drop from n to a



$$\bar{V}_{ab} = \bar{V}_{an} + \bar{V}_{nb}$$

Let V_p is the effective value of ~~voltage~~ voltage of each phase (balanced system).

$$\bar{V}_{nb} = V_p \angle -120^\circ$$

and $\bar{V}_{nc} = V_p \angle 120^\circ$

$$= V_p \angle -240$$

$$V_{na} = V_p \angle 0^\circ$$

$$V_{nb} = V_p \angle -120^\circ$$

$$V_{nc} = V_p \angle 120^\circ$$

$$= V_p \angle -240^\circ$$

$$\bar{V}_{ab} = -V_p + V_p \angle -120^\circ$$

$$= -V_p + V_p [\cos(-120^\circ) + j\sin(-120)]$$

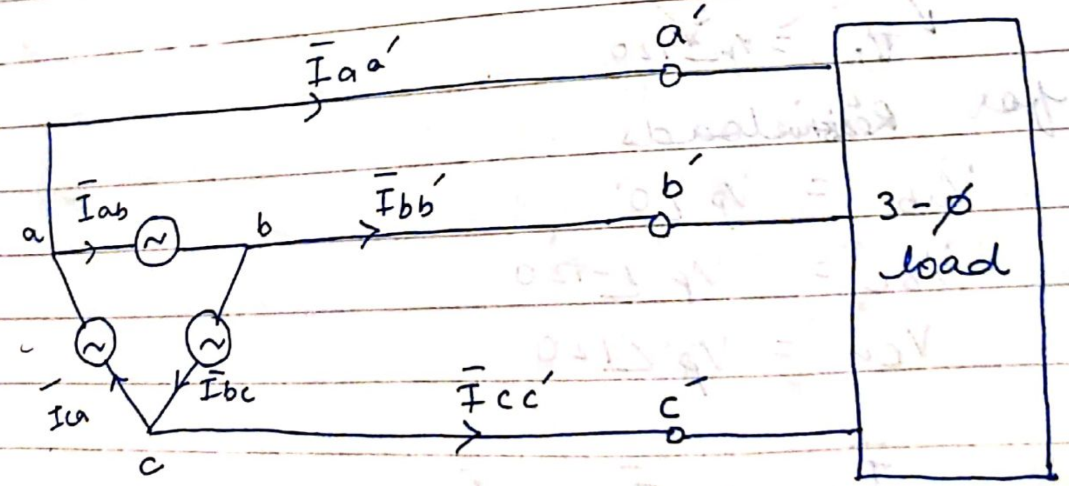
$$= \sqrt{3} V_p \angle -150^\circ$$

$$\bar{V}_{bc} = \bar{V}_{bn} + \bar{V}_{nc}$$
$$= -\bar{V}_{nb} + \bar{V}_{nc}$$
$$= -V_p \angle -120 + V_p \angle 120^\circ$$
$$= \sqrt{3}\, V_p \angle 90^\circ .$$
$$= \sqrt{3}\, V_p \angle -270^\circ$$

$$\bar{V}_{ca} = \bar{V}_{cn} + \bar{V}_{na}$$
$$= -\bar{V}_{nc} + \bar{V}_{na}$$
$$= -V_p \angle 120^\circ + V_p \angle 0^\circ$$
$$= \sqrt{3}\, V_p \angle -30^\circ .$$

17th Nov '03

Current and Voltage relationship for a Δ-connected Three phase system :-



For a Δ connected system,
Line voltage (V_L) = phase voltage (V_P)

$$\boxed{V_L = V_p}$$

apply KCL at pt 'a'

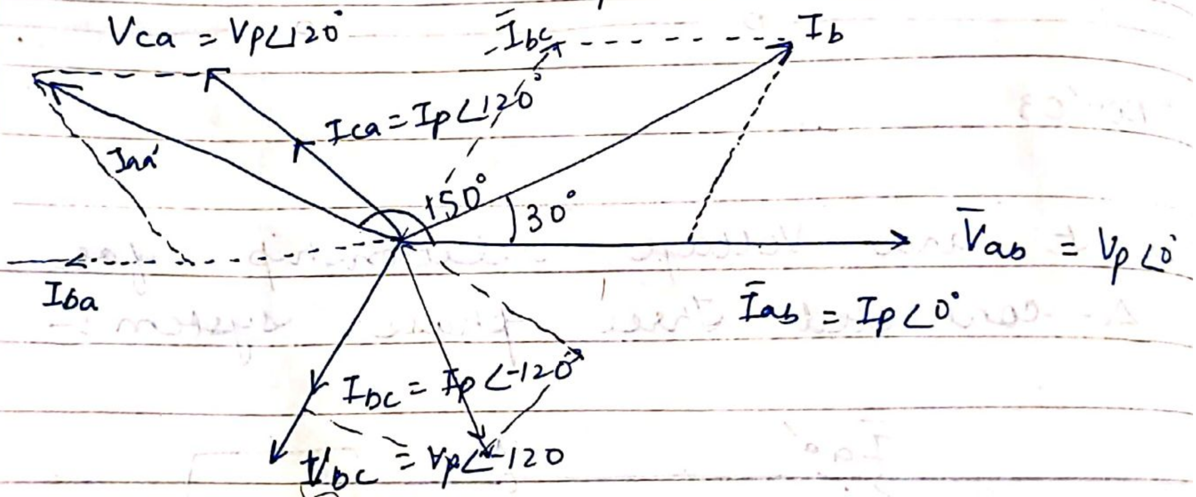
$$\bar{I}_{ca} = \bar{I}_{aa'} + \bar{I}_{ab}$$

$$\Rightarrow \bar{I}_{aa'} = \bar{I}_{ca} - \bar{I}_{ab} = \bar{I}_{ca} + \bar{I}_{ba}$$

$$\bar{I}_{ab} = I_p \angle 0^\circ$$

$$I_{bc} = I_p \angle -120$$

$$I_{ca} = I_p \angle 120 = I_p \angle -240$$



for Resistive loads

$$V_{ab} = V_p \angle 0^\circ$$

$$V_{bc} = V_p \angle -120$$

$$V_{ca} = V_p \angle 120$$

$$\bar{I}_{aa'} = \bar{I}_{ca} + \bar{I}_{ba}$$

$$= I_p \angle 120^\circ - I_p \angle 0^\circ$$

$\bar{I}_{aa'} = I_p(1\angle 120^\circ - 1\angle 0^\circ)$

$\bar{I}_{aa'} = \sqrt{3}\, I_p \angle 150^\circ$ $\qquad$ $I_{aa'} \to$ line current

$$\boxed{I_L = \sqrt{3}\, I_p}$$

apply KCL at point b.

$$\bar{I}_{ab} = \bar{I}_{bb'} + \bar{I}_{bc}$$

$$\bar{I}_{bb'} = \bar{I}_{ab} - \bar{I}_{bc}$$
$$= I_p\angle 0 - I_p\angle -120^\circ$$
$$= I_p(1\angle 0^\circ - 1\angle -120^\circ)$$
$$= \sqrt{3}\, I_p \angle 30^\circ$$

apply KCL at point c :-
$$I_{bc} = \bar{I}_{ca} + \bar{I}_{cc'}$$

$$\bar{I}_{cc'} = \bar{I}_{bc} - \bar{I}_{ca}$$
$$= I_p\angle -120^\circ + I_p\angle 120^\circ$$
$$= I_p(1\angle -120 + 1\angle 120^\circ)$$
$$= \sqrt{3}\, I_p \angle -90^\circ.$$

for a single phase system

$$P(t) = VI\cos\theta + VI\cos(2\omega t - \theta)$$

for a three phase system, the instantaneous power in each phase will be given by:-

$$P_a(t) = V_p I_p \cos\theta + V_p I_p \cos(2\omega t - \theta)$$

$$P_b(t) = V_p I_p \cos\theta + V_p I_p \cos(2\omega t - \theta - 120°)$$

$$P_c(t) = V_p I_p \cos\theta + V_p I_p \cos(2\omega t - \theta - 240)$$

θ = angle between V_p and I_p. It is the impedance angle of load.

Total instantaneous power in all the 3 phases;

$$P(t) = P_a(t) + P_b(t) + P_c(t).$$

$$= 3V_p I_p \cos\theta + [V_p I_p \cos(2\omega t - \theta) + V_p I_p \cos(2\omega t - \theta - 120°) + V_p I_p \cos(2\omega t - \theta - 240°)$$

Put $\omega t = 0$

$$= 3V_p I_p \cos\theta + \{V_p I_p [\cos(+\theta) + \cos(\theta + 120) + \cos(\theta + 240)]$$

$$= 3V_p I_p \cos\theta + V_p I_p [\cos\theta + \cos\theta\cos 120 - \sin\theta\sin + \cos\theta\cos 240 - \sin\theta$$

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$$= 3V_p I_p \cos\theta + V_p I_p \left[\cos\theta + -\frac{\cos\theta}{2} - \sin\theta \frac{\sqrt{3}}{2} - \frac{\cos\theta}{2} + \sin\theta \frac{\sqrt{3}}{2} \right]$$

$$\underbrace{}_{= 0}$$

$$= 3V_p I_p \cos\theta$$

$$\therefore P(t) = 3 V_p I_p \cos\theta$$

$$3\text{-}\phi \text{ average power} = \frac{1}{T}\int_0^T \left(P_a(t) + P_b(t) + P_c(t)\right) dt$$

$$\boxed{P_{3-\phi} = 3 V_p I_p \cos\theta}$$

If The value is line value if not mentioned:

Y

$$P_{3-\phi} = 3 V_p I_p \cos\theta$$

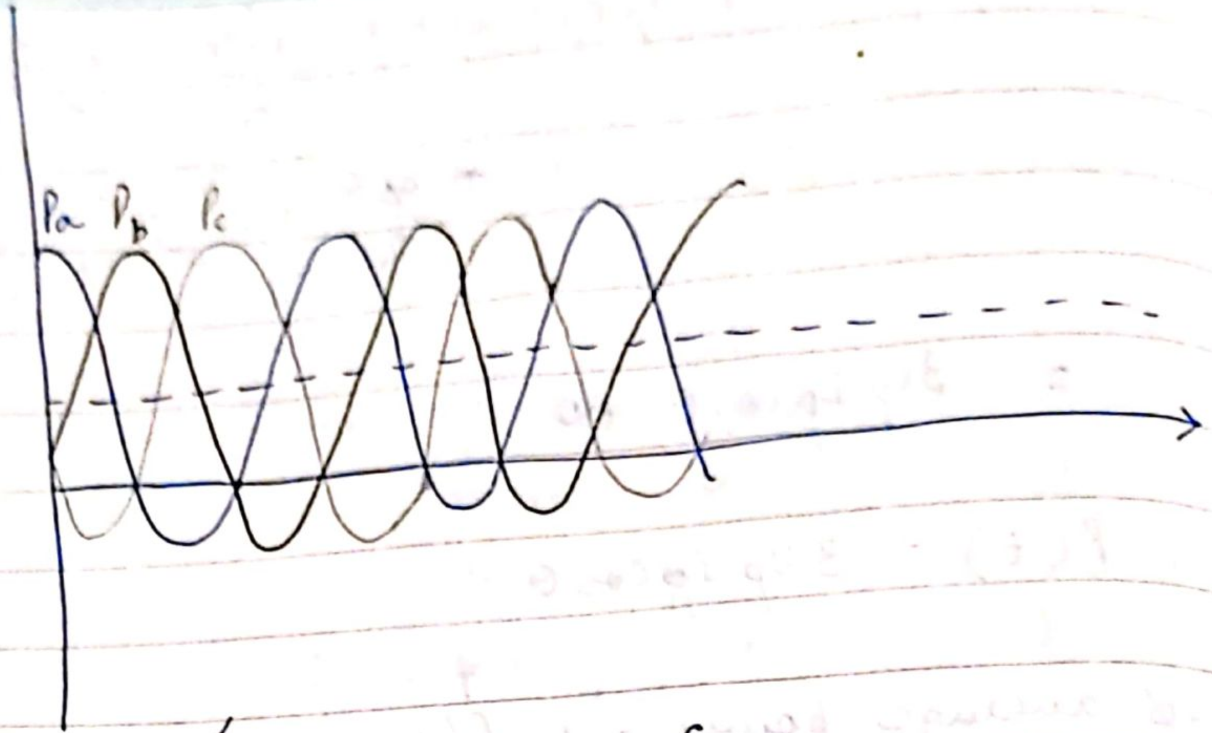
$$= 3 \frac{V_L}{\sqrt{3}} \times I_L \cos\theta$$

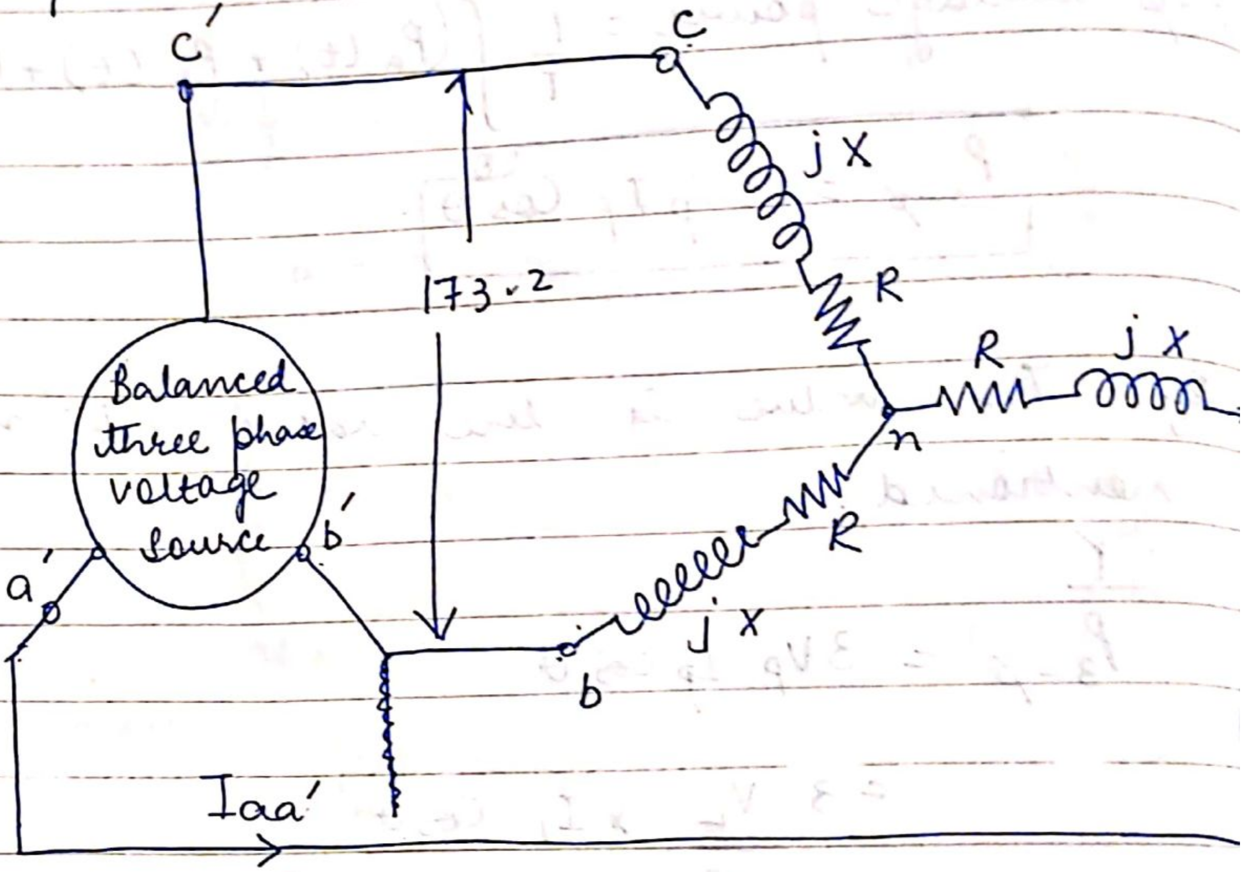
$$\boxed{P_{3-\phi_Y} = \sqrt{3} V_L I_L \cos\theta}$$

Δ

$$P = 3 \times V_L \times \frac{I_L}{\sqrt{3}} \cos\theta$$

$$\boxed{P_{3-\phi_\Delta} = \sqrt{3} V_L I_L \cos\theta}$$





$$V_L = 173.2\,V$$

$$\bar{Z}_a = \bar{Z}_b = \bar{Z}_c = 10\angle 45^\circ \;;$$

$$V_p = \frac{V_L}{\sqrt{3}} = \frac{173.2}{\sqrt{3}} = 100\,V \;;$$

$$\boxed{\begin{aligned} V_{an} &= 100\angle 0 \\ \bar{V}_{bn} &= 100\angle -120^\circ \\ V_{cn} &= 100\angle 120^\circ \end{aligned}}$$

$I_{an} = V_{an}/\bar{Z}_a = 10\angle -45$

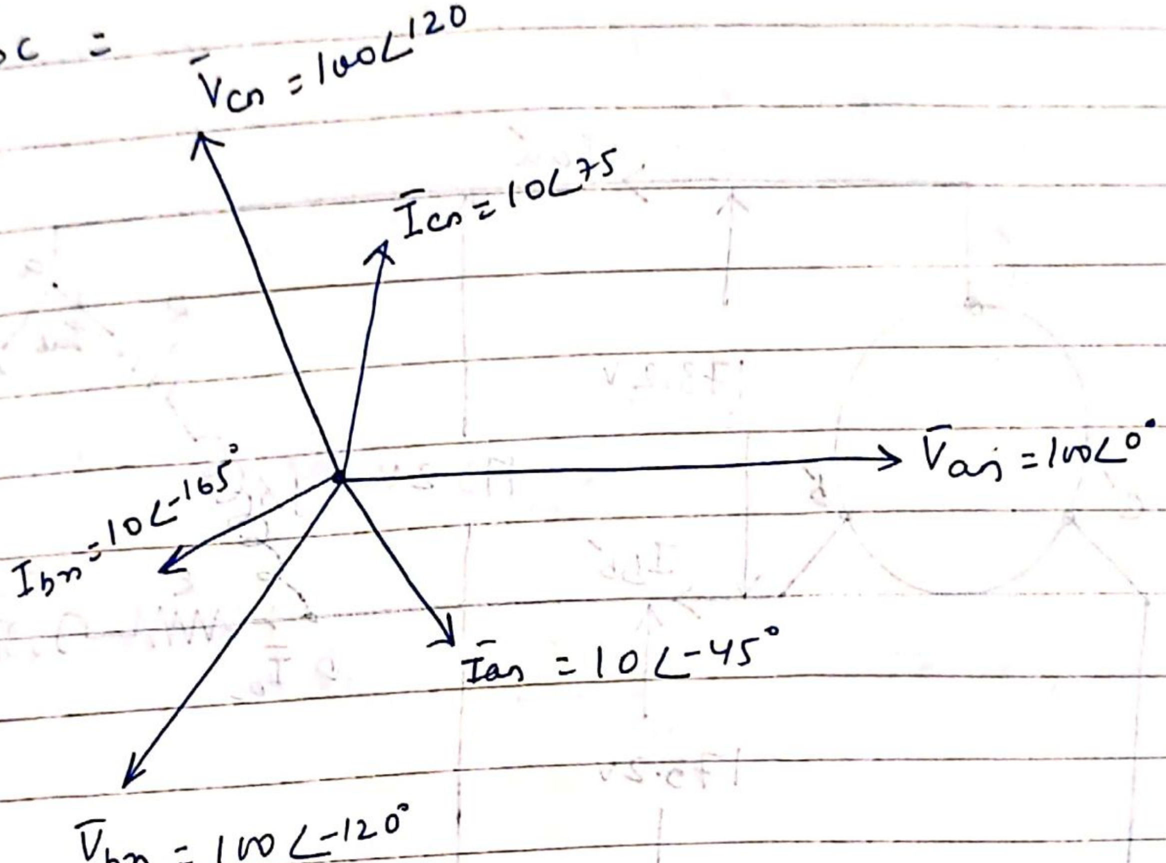
$\bar{I}_{bn} = \bar{V}_{bn}/\bar{Z}_b = \frac{100\angle -120}{10\angle 45} = 10\angle -165^\circ$

$\bar{I}_{cn} = \bar{V}_{cn}/\bar{Z}_c = \frac{100\angle 120}{10\angle 45} = 10\angle 75^\circ$

$$I_p = \frac{\bar{V}_p}{Z} = \frac{100}{10\angle 45} = 10\angle -45^\circ.$$

$\bar{V}_{ab} = 173.2\angle 0^\circ$

$\bar{V}_{bc} =$



av power/phase

$P_a = V_p I_p \cos\theta$

$= 100 \times 10 \times \cos 45 = 707.11$ W

$P_{3\phi} = 3 \times 707.11$

$= 2121$ W

LP-13

$P_{3-\phi} = \sqrt{3}\, V_L\, I_L \cos\theta$

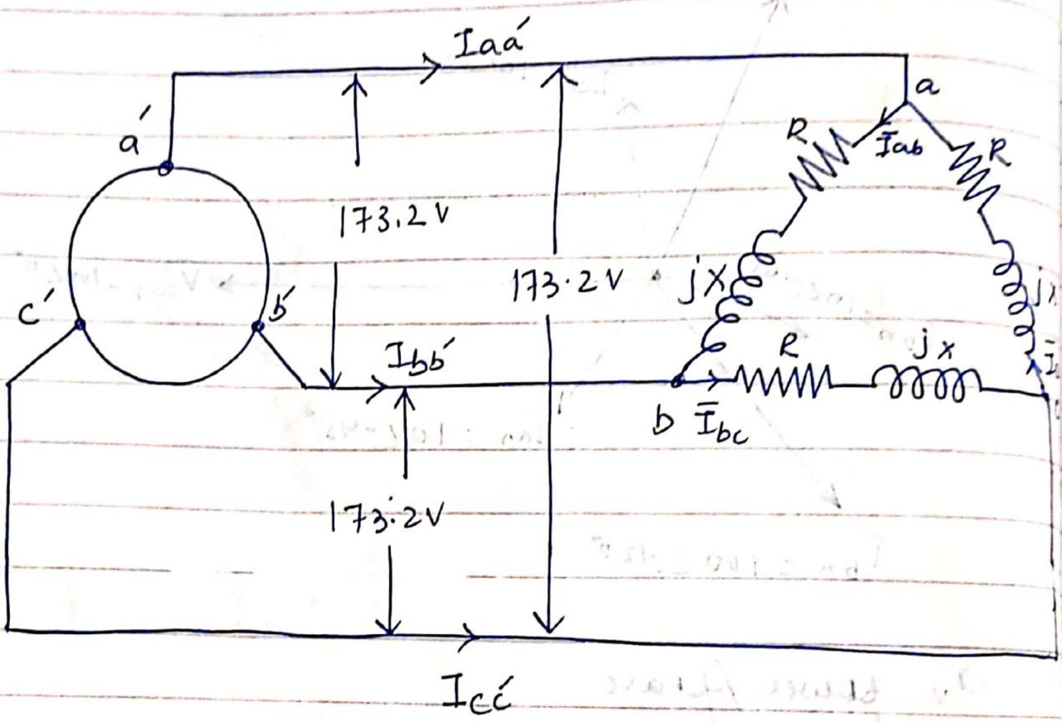
$= \sqrt{3} \times 173.2 \times 10 \times \cos 45$

$= 2121.25\ W$

$P_{3-\phi} = I_{an}^2 R$

$= 10^2 \times 7.07 \times 3$

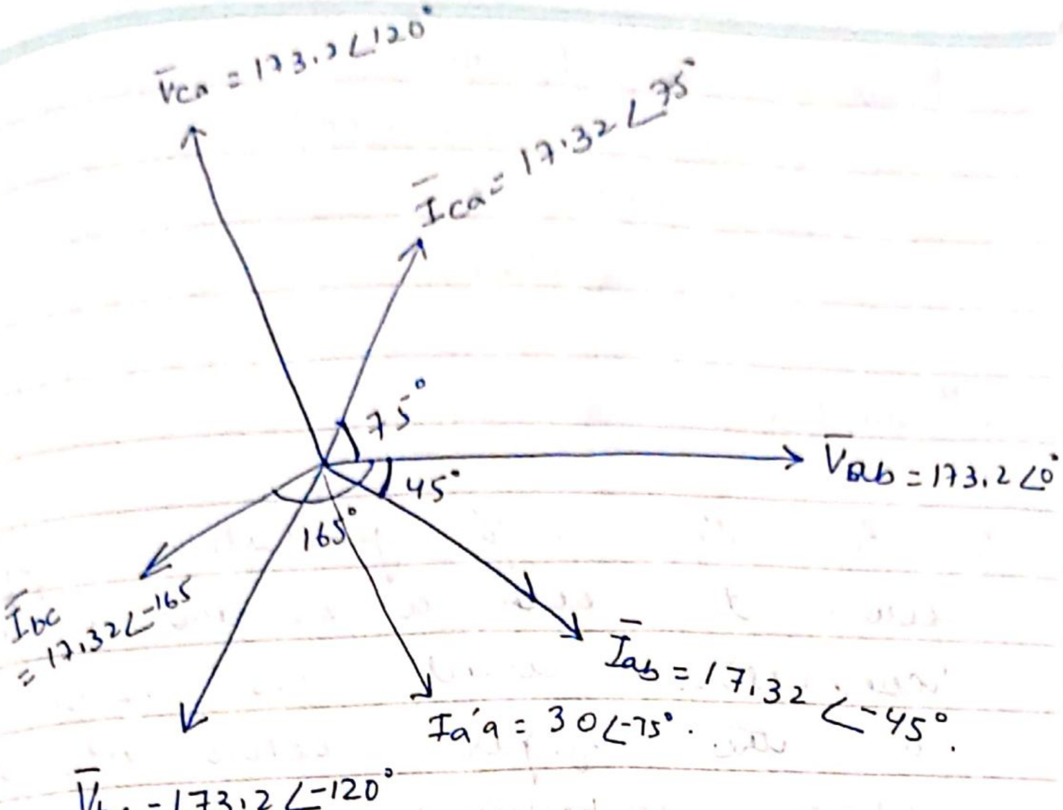
$= 2121\ W.$



$\bar{V}_{ab} = 173.2 \angle 0^\circ$

$\bar{V}_{bc} = 173.2 \angle -120^\circ$

$\bar{V}_{ca} = 173.2 \angle 120^\circ.$



Now, $\bar{I}_{ab} = \bar{V}_{ab}/\bar{Z}_a = \dfrac{173.2\angle 0°}{10\angle 45°} = 17.32\angle -45°$

$\bar{I}_{bc} = \bar{V}_{bc}/\bar{Z}_b = \dfrac{173.2\angle -120}{10\angle 45°} = 17.32\angle -165°$

$\bar{I}_{ca} = \bar{V}_{ca}/\bar{Z}_c = \dfrac{173.2\angle 120}{10\angle 45} = 17.32\angle 75°$

apply KCL at pt 'a'.

$\bar{I}_{a'a} + \bar{I}_{ca} = \bar{I}_{ab}$

$I_{a'a} = \bar{I}_{ab} - \bar{I}_{ca}$

$= 17.32\angle -45° - 17.32\angle 75°$

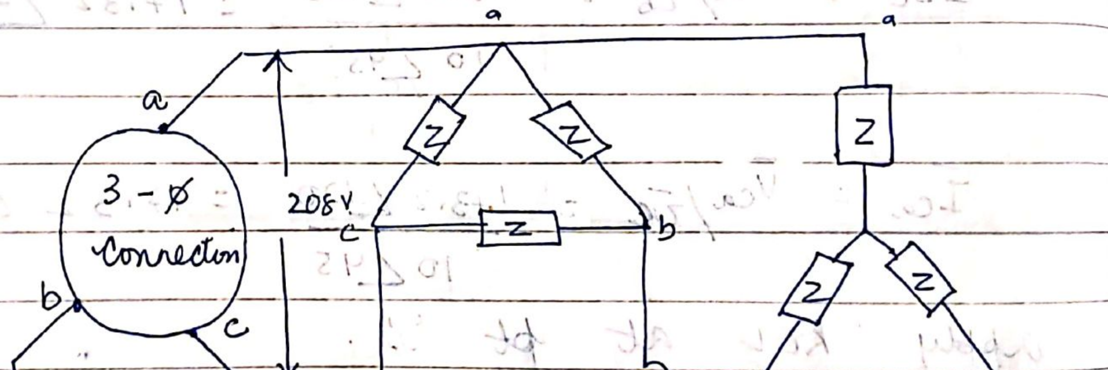
$= 30\angle -75°$

find $I_{b'b}$ and $I_{c'c}$

$$P_{3-\phi} = \sqrt{3}\, V_L \times I_L \cos\theta$$
$$= \sqrt{3} \times 173.2 \times 30 \times \cos 45$$
$$= 6363.77$$
$$= 3(2121)$$

18th Nov'03 :-

Q A 208 V, 3-ϕ generator supplies power to both a Δ and a Y connected load as shown in the figure below. All the phase impedances are equal & specifically equal to $5 + j\,8.66$. Compute the total generator current $\bar{c}$ flow in line a.



for Δ connected loads;

phase Voltage is $= 208V$

$$I_p = \frac{208}{5+j8.66} = 20.8\angle{-60^\circ}$$

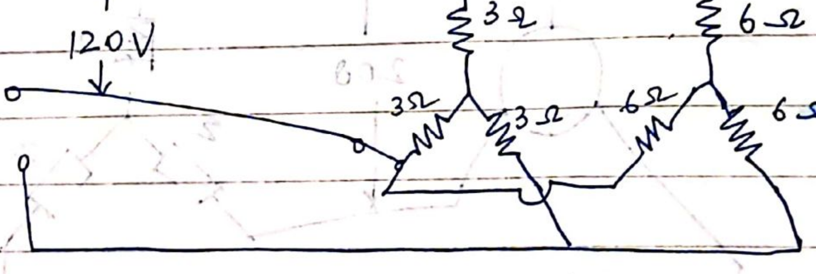
for Y- connected load;

$$I_p = \frac{\frac{208\angle 0^\circ}{\sqrt{3}}}{5+j8.66} = 12.0\angle{-60^\circ}$$

Total line current $= \sqrt{3}\,(20.8\angle{-60^\circ} + 12\angle{-60^\circ})$

$= 48\angle{-60^\circ}$ (wrong).

Q In a 120V 3-ϕ ckt find the total line current to 2 balanced resistive star connected loads, one of 3Ω resistors & the other of 6Ω resistors.



A.C Circuits

Tutorial Sheet no. 5.

P5.1: Perform the source conversion on a voltage source of $9\angle 15^\circ$ V with a series impedance of $2\angle 45^\circ\ \Omega$.

P5.2: A voltage source has a $100\sin(100t - 30^\circ)$ V open-circuit voltage and a $80\angle 60^\circ\ \Omega$ internal impedance. Perform a source conversion. Specify element values.

P5.3: Convert a voltage source in Fig P5.3a to an equivalent current source, and the current source in Fig P5.3b to an equivalent voltage source. In each case, draw the schematic diagram of the equivalent source and label its impedance in polar form.

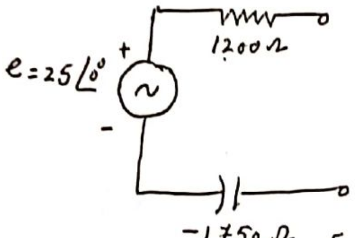


Fig P5.3a

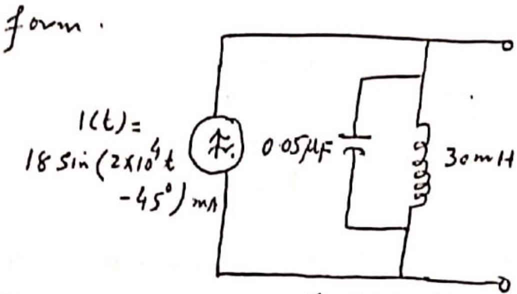


Fig P5.3b

P5.4: Find the current in the inductor in Fig P5.4 by converting the current source to an equivalent voltage source.

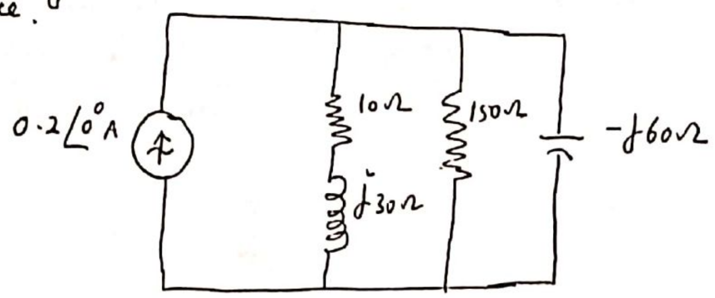


Fig P5.4

5: Find mesh currents I_1 and I_2 in the circuit of Fig P5.5.

P5.6: Choose loop currents for the circuit of Fig P5.5 such that I_2 is the only current through the 2Ω resistor, and then find I_2.

P5.7: Show a circuit that corresponds to the following mesh equations

$$(12-j2)I_1 - (6-j4)I_2 = 5-j7$$
$$-(6-j4)I_1 + (14+j9)I_2 = -3+j2$$

P5.8: Find the current flowing up in 2Ω resistor in circuit of Fig P5.8.

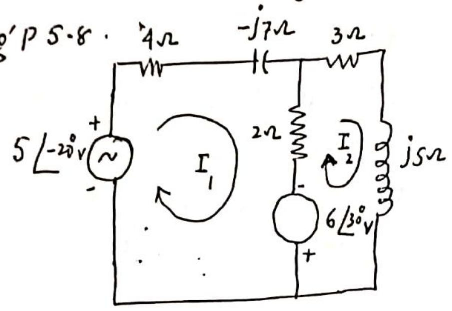


Fig P5.5

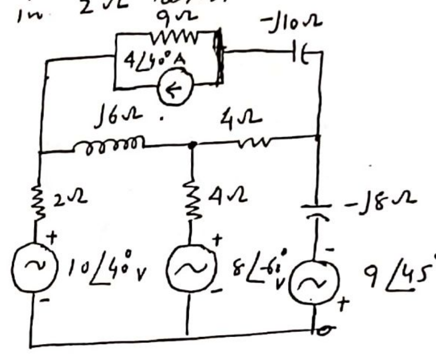


Fig P5.8.

P5.9: Select loop currents in the circuit of Fig P5.8 such that only one loop current flows through the 4-Ω resistor in the centre branch. Then find this current.

P5.10: Use mesh analysis to find the polar form of the current in the resistor in Fig P5.10.

P5.11: Use mesh analysis to find the polar form of the voltage across the 40Ω resistor in Fig P5.11.

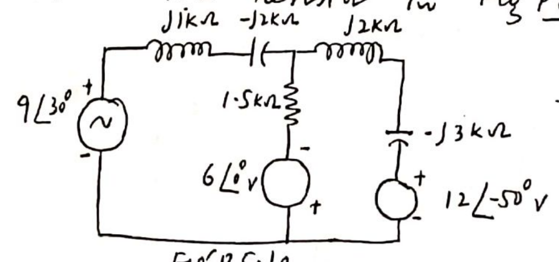


Fig P5.10

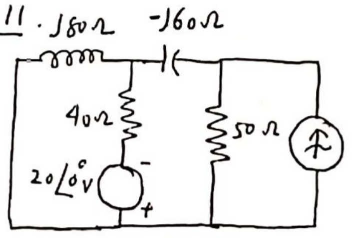


Fig P5.11

Scanned with CamScan

5.12: Use nodal analysis to find the polar form of the voltage across the inductor in Fig P5.12. Draw + and − signs to show the phase reference of the calculated voltage.

P5.13: Use nodal analysis to find the polar form of the current in the capacitor in Fig P5.13. Draw an arrow to show the phase reference of the calculated current.

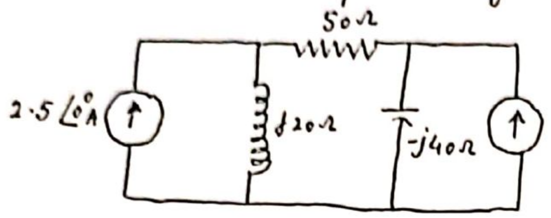


Fig P5.12

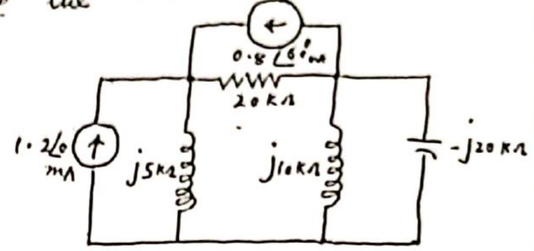


Fig P5.13

P5.14 : Find the node voltages $\bar{V}_1$ and $\bar{V}_2$ in the circuit of Fig P5.14.

P5.15 : In the circuit of Fig P5.5 use nodal analysis to find the voltage drop, top to bottom, across the branch containing the 2-Ω resistor and the $6\angle 30°$-V voltage source. Use only one equation.

P5.16 : Show a circuit with just one current source that corresponds to the nodal equations:

$$(6+j5)\bar{V}_1 - 4\bar{V}_2 = 4\angle -45°$$
$$-4V_1 + (8-j7)\bar{V}_2 = 4\angle 135°$$

P5.17 : For the circuit shown in Fig P5.8, replace the $-j10$-Ω capacitor by an open circuit and then use nodal analysis to find the voltage drop, top to bottom, across the branch containing the 2-Ω resistor and the $10\angle 40°$-V source.

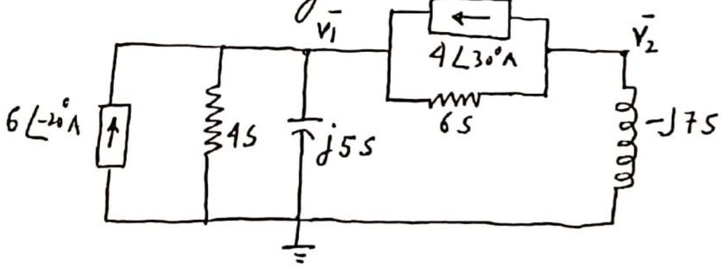


Fig P 5.14

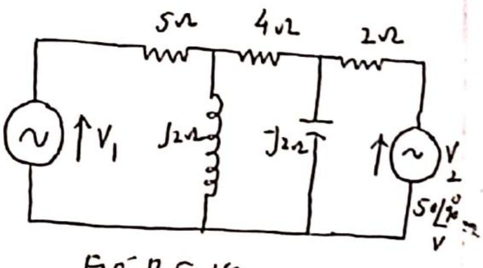


Fig P5.18

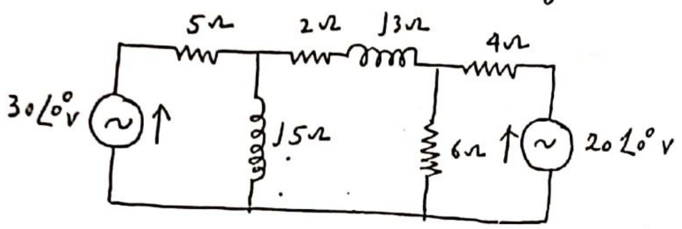
18: In the network of Fig P5.18, find the voltage V_1 such that the current in the 4 ohm resistor is zero. Use (a) mesh analysis (b) nodal analysis (Select one end of the resistor as the reference node.

$[95.4 \angle -23.2^\circ \text{ V}]$

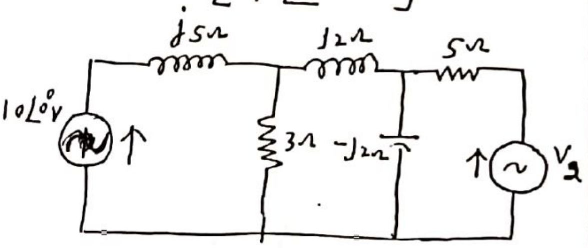
P5.19: Referring to the circuit of Fig' P5.18, source $\bar{V}_1 = 50\angle 0^\circ$ V and $\bar{V}_2$ is unknown. Find $\bar{V}_2$ such that the current in the 4 Ω resistor is zero. Use (a) mesh analysis (b) nodal analysis $[26.2 \angle 113.2^\circ \text{ V}]$

P5.20: In the network of Fig' P5.20, find the power in the 6 Ω resistor by nodal method. [39.6 W]

P5.21: In the network of Fig' P5-21, find the voltage $\bar{V}_2$ such that its current will be zero $[4 \angle 180^\circ \text{ V}]$



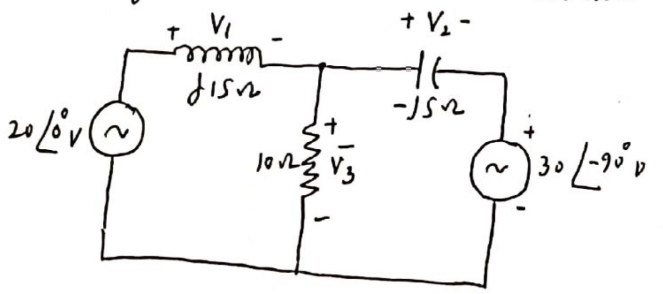
Fig' P5.20



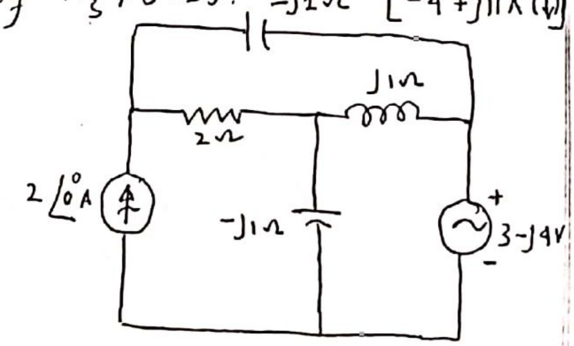
Fig' P5.21

P5.22: Assuming that both sources in Fig' P5.22 operate at the same frequency, find (a) $\bar{V}_1$, (b) $\bar{V}_2$ (c) $\bar{V}_3$ using nodal analysis. $[33.9\angle 81.9^\circ, 15.62\angle -13.32^\circ, 36.9\angle -65.7^\circ \text{ V}]$

P5.23: Use the node method to find the current through the voltage source in the circuit of Fig' P5.23. $[-4 + j11 \text{ A (V)}]$



Fig' P5.22



Fig' P5.23

4: A Δ network has impedances $Z_1 = 0.3\,\Omega$, $Z_2 = 0.4\,\Omega$ and $Z_3 = 0.5 - j0.5\,\Omega$. Find the corresponding Y network.

P5.25: In the circuit of Fig P5.25 find I using either Y to Δ or a Δ-Y conversion.

P5.26: Use a Δ-to-Y or a Y-to-Δ conversion to find the current I in the circuit of Fig P5.26

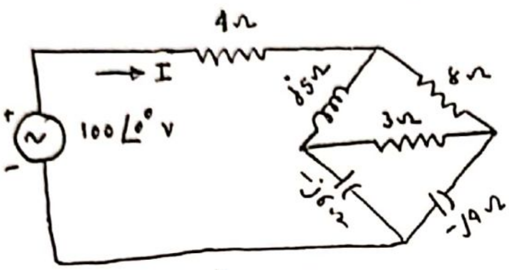


Fig P5.25

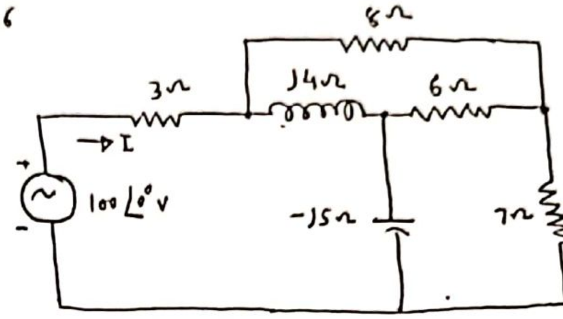


Fig P5.26

P5.27 In the circuit of Fig P5.25 what impedance substituted for the $-6\,\Omega$ capacitive reactance produces zero current through $3\,\Omega$ resistor.

P5.28: The bridge circuit of Fig P5.28 balances for $Z_1 = 10\,\Omega$, $Z_2 = 4\angle 30^\circ\,\Omega$, $Z_3 = 6\,\Omega$, $\omega = 100$ rad/s and a Z_x comprising of two elements in series. What are they and what are their values?

P5.29: If the capacitance comparison bridge of Fig P5.29 is in balance with $R_1 = 50\,k\Omega$, $R_2 = 2\,k\Omega$, $R_3 = 1\,k\Omega$ and $C_s = 2\,\mu F$, what are C_x and R_x?

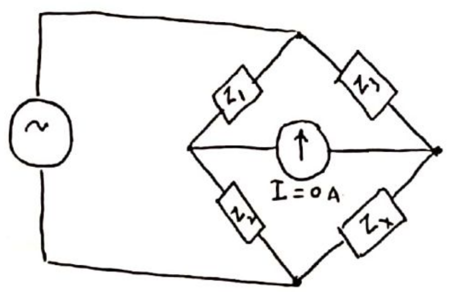


Fig P5.28

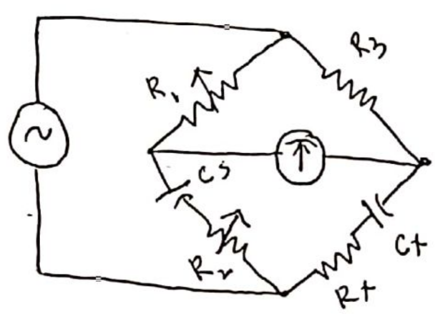


Fig P5.29

P5.30: In the circuit shown in Fig P5.30 find the Thevenin and Norton equivalent circuits looking in at terminals a-b

P5.31: A circuit has configuration depicted in Fig P5.31
(a) Find the equivalent impedance appearing to the right of points a b.
(b) Determine the value of the reactance X which makes the source current in phase with the source voltage.
(c) Should the reactance X of part (b) be inductive or capacitive? Find the required value of L or C.
(d) Compute the effective value of the source current for the condition described in part (b).

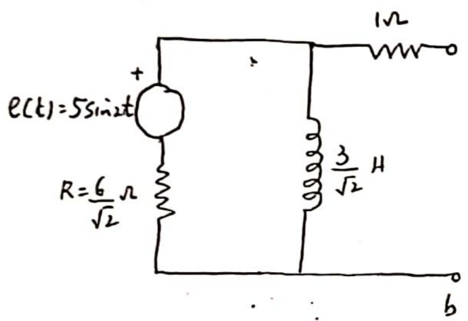


Fig P5.30

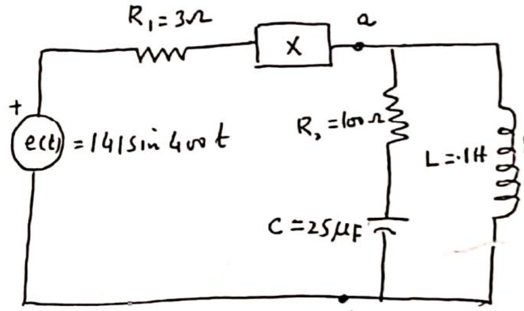


Fig P5.31

Tutorial Sheet - 6

A.C. Power

P6.1: Given a circuit with an applied voltage $v(t) = 14.14 \cos \omega t$ (V) and resulting current $i(t) = 17.1 \cos(\omega t - 14.05)$ (mA), determine the complete power information. [P = 117 mW, Q = 97.3 mvar (inductive), pf = 0.770 lagging]

P6.2: Given a circuit with an applied voltage $v(t) = 340 \sin(\omega t - 60^\circ)$ (V) and a resulting current $i(t) = 13.3 \sin(\omega t - 48.7^\circ)$ (A), determine the complete power information. [P = 2217 W, Q = 443 var (capacitive), pf = 0.981 leading]

P6.3: What is the power factor of a circuit with a $4\angle{-30^\circ}\ \Omega$ impedance? If the rms applied voltage is 40 V, what is the average power absorbed. [0.866 leading, 346.4 W]

P6.4: If a circuit has a $30 + j40\text{-}\Omega$ input impedance and an ~~200V applied voltage~~ input current of 4 A, what is the average power absorbed? [480 W]

P6.5: What is the power factor of a two terminal circuit with a $10\angle 45^\circ\ \Omega$ input impedance? If the input current is 2 A, what is the average power absorbed? [0.707 lagging, 28.3 W]

P6.6: A series circuit comprising of a resistor and an inductor dissipates 10 W when the combination is connected to a 115 V, 60 Hz source. If the power factor is 0.6 lagging, what are the values of resistance and inductance? [476.1 Ω, 1.68 H]

P6.7: A resistor in series with a capacitor dissipates 5 W when the combination is connected to a 110 V, 1000-Hz source. If the power factor is 0.5 leading, what are the resistance and capacitance? [605 Ω, 0.152 μF]

P6.8: What are the vars consumed by a circuit with $60 + j20\text{-}\Omega$ input impedance and a 10-A input current? [2 kvar]

P6.9: A circuit with a $30\angle 50^\circ\ \Omega$ input impedance has a 4-A input current. What are the vars consumed? [367.7 var]

The combination of two elements in series consumes 50 var when connected to a 100V, 60-Hz source. If the reactive factor is 0.5, what are the two components and their values?
[86.6 Ω, 0.133 H]

P6.11: A load consumes 400 var when energized from a 200V source. If the reactive factor is 0.4, what current does the load draw and what is the load impedance? [5A, $40\angle 23.6^\circ$ Ω]

P6.12: A 120V, 60 Hz voltage source energises a 0.1H coil having 10 Ω of resistance. Find the power components.
[94.7W, 357 var, 369 VA]

P6.13: A 120V, 60-Hz voltage source energises the parallel combination of 10 μF capacitor and a 1-H coil having 100 Ω of resistance. Find the power components and the input current.
[9.47W, 18.6 var, 174 mA]

P6.14: A 20-Ω resistance is in parallel with a 0.1H coil having 10 Ω of resistance. If the parallel combination has a current input of 10A at ω = 1000 rad/s, what are the power components?
[1890W, 367 var, 1925 VA]

P6.15: A 10 Ω resistor and an unknown impedance in series consume 500 VA at 0.6 leading power factor. If the current is 4A, find the unknown impedance in polar form.
[$26.5\angle -70.7$ Ω]

P6.16: A two-element series circuit has average power of 940 W and power factor 0.707 leading. Determine the circuit elements if the applied voltage is $v(t) = 99\cos(6000t + 30^\circ)$ V.
[R = 2.6 Ω, C = 64.1 μF]

P6.17: The parallel circuit shown in Fig P6.17 has a total average power of 1500 W. Obtain the total power information.
[1500W, 2471 var, Pf = 0.519 lagging]

P6.18: Determine the average power in the 15 Ω and 8 Ω resistances in Fig P6.18, if the total average power is 2000W.
[723W, 1277W]

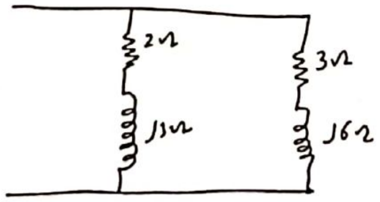


Fig P6.17.

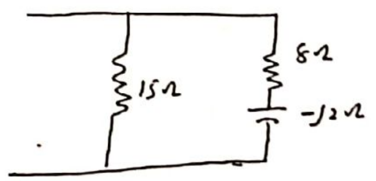


Fig P6.18

19: Obtain the complete power information for the following parallel-connected loads:

load #1 5 KW, pf = 0.80 lagging
load #2 4 KVA, 2 KVAR (capacitive)
load #3 6 KVA, pf = 0.90 (lagging)

[14.535 KVA; pf = 0.954 lagging]

P6.20: Obtain the complete power information for the following parallel-connected loads:

load #1 200 VA, pf = 0.7 (lagging)
load #2 350 VA, pf = 0.5 (lagging)
load #3 275 VA, pf = 1.00

[590 W; 444 VAR; 0.799 (lag)]

P6.21: How many KVars must be supplied by parallel capacitors to 25 KVA load having a 0.6 lagging power factor in order to increase the overall power factor to 0.9 lagging?
[12.7 KVar]

P6.22: A 4500 VA load at a power factor of 0.75 lagging is supplied by a 60 HZ source at effective voltage 240V. Determine the parallel capacitance in microfarads necessary to improve the power factor to (a) 0.90 lagging, (b) 0.90 leading.
[61.8 μF, 212 μF]

P6.23: A 240-V, 60 HZ source supplies 10 KVA to a load having 0.7 lagging power factor. Find the parallel capacitance required to improve the overall power factor to 0.9 lagging. Also find the corresponding decrease in line current. [173 μF, 9.26 A]

P6.24: The addition of a 20-KVar capacitor bank improved the power factor of a certain load to 0.9 lagging. Determine P and Q before the addition of capacitors, if the final apparent power is 185 KVA
[166.5 KW; 100.6 KVAR]

P6.25: For a load energised by a 240V, 60 HZ source, an added (parallel) capacitance of 10 μF improves the power factor from 0.7 lagging to 0.85 lagging. What is the source current both before and after the capacitance is added? [3.23 A, 2.66 A]

P6.26: Find the capacitance C necessary to improve the power factor to 0.95 lagging in the circuit shown in Fig P6.26, if the effective voltage of 120 V has a frequency of 60 HZ.
[28.6 μF]

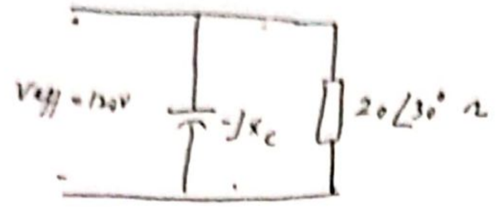


Fig P6.26.

P6.27: A 25-KVA load with power factor 0.80 lagging has a group of resistive heating elements added at unity power factor. How many KW do these units take, if the new overall power factor is 0.85 lagging. [4.2 KW]

P6.28: A 1000-W electric motor is connected to a 200V rms, 60-HZ ac source and the result is a lagging power factor of 0.8.
a) If a 28-μF capacitor is placed in parallel with the motor, what is the resulting pf. (b) what value of C will cause the resulting pf. to a unity value. [0.9, current reduction 16% ; 49.7μF]

P6.29: A load with lagging power factor of $1/\sqrt{2}$ draws 11.5 KW of power when connected across $115\sqrt{2}$ V peak, 60 HZ line.

(a) what is the line current?

(b) what size capacitor will bring the overall P.f. to unity? what is the resulting line current?

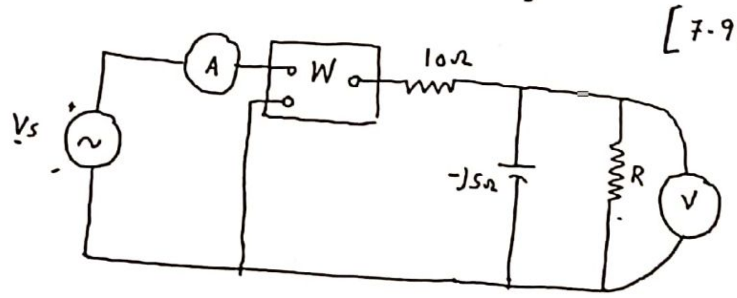
[$\frac{200A}{\sqrt{2}}$; 2300 μF, $\frac{141.42A}{\sqrt{2}}$]

P6.30: For the circuit shown in Fig P6.30, the wattmeter reads 1090W, the ammeter reads 9.43 A and voltmeter reads 40V.

(a) what is R?

(b) what is peak value of source voltage Vs?

[7.97Ω, 170.5V]



Tutorial Sheet - 7

Three Phase Circuits

P7.1: What is the phase sequence for a three-phase Y-circuit in which $V_{an} = 2300\angle 30^\circ$ V, $V_{bn} = 2300\angle 150^\circ$ V and $V_{cn} = 2300\angle -90^\circ$ V?
[ACB]

P7.2: What is the phase sequence for a three-phase Y circuit in which two of the phase voltages are $V_{an} = 230\angle 45^\circ$ and $V_{cn} = 230\angle 165^\circ$ V?
[ABC]

P7.3: A three-phase Y circuit with a positive phase sequence has one phase voltage of $V_{bn} = 200\angle 40^\circ$ V. Find the line voltages V_{ab}, V_{bc} and V_{ca}. [$346\angle 190^\circ$ V, $346\angle 70^\circ$ V, $346\angle -50^\circ$ V]

P7.4: A balanced three-wire three-phase circuit with a positive phase sequence has one line current of $I_a = 100\angle -30^\circ$ A. Find the two other line currents. [$100\angle -150^\circ$ A, $100\angle 90^\circ$ A]

P7.5: Find the phase currents I_{AB}, I_{BC} and I_{CA} for a balanced Δ load to which one line current is $I_A = 35\angle -20^\circ$ A. The phase sequence is negative. [$20.2\angle -50$ A, $20.2\angle 70^\circ$ A, $20.2\angle -170^\circ$ A]

P7.6: A balanced Δ-load has one phase current of $I_{CB} = 5\angle -20$ A. Find the other phase currents I_{BA} and I_{AC} and the three line currents, given that phase sequence is positive.
[$5\angle -140$, $5\angle 100$; $8.66\angle -110^\circ$, $8.66\angle 130^\circ$, $8.66\angle 10^\circ$]

P7.7: What are the line currents to a Δ load if one phase current is $20\angle 10^\circ$ A and the phase sequence is positive?
[$34.6\angle -20$, $34.6\angle -140^\circ$, $34.6\angle 100^\circ$]

P7.8: Assume a positive phase sequence and find the line currents to a three-phase Y load in which each impedance is $40\angle -30^\circ$ Ω and $V_{AN} = 100\angle -40^\circ$ V.
[$2.5\angle -10$, $2.5\angle -130^\circ$, $2.5\angle 110^\circ$]

P7.9: A balanced Y-Y three-phase circuit has 130 V rms phase voltages and per phase impedance of $Z = 12 + j12$ Ω. Find line currents and the total power absorbed by the load. [$7.66\angle -45$, $7.66\angle -165$, $7.66\angle 75^\circ$; 2110 W]

P7.10: A balanced Y-Y three phase circuit has 210 V, 60 Hz line voltages. Suppose that the load absorbs a total of 3 kW of power at 0.85 pf. (lagging). Find the per phase impedance.
[$10.5 + j6.53$ Ω]

total line

to two balanced resistive Y loads, one of 3-Ω resistors and the other of 6-Ω resistors. [34.6 A]

P7.12: A balanced three-phase three-wire system has a line voltage of 440 V rms and feeds two balanced Y-connected loads. One is an inductive load with $7+j2$ Ω per phase, and the other is a capacitive load with $5-j2$ Ω per phase. Find: (a) the rms current (line) provided by the source; (b) the average power delivered to inductive load; (c) the average power generated by the source.

[77.8 A ; 25.6 KW ; 58.9 KW].

P7.13: The balanced three-phase circuit of Fig P7.13 operates with $V_{BC} = 100\angle 20^\circ$ Vrms with positive phase sequence. The total load draws 3 KW at pf = 0.8 lagging. If $R_W = 0.6\,\Omega$, find: (a) the total power lost in the line resistances; (b) V_{ab}. [843.75 W, 118.77 $\angle 133.47^\circ$]

P7.14: Refer to balanced Y-Y circuit of Fig P7.13 and let $V_{an} = 4600\angle 0^\circ$ Vrms with (+) phase sequence. Let the source supply $P = 240$ kW and $Q = 60$ KVAR with $R_W = 3.2\,\Omega$. Find: (a) V_{AN}; (b) I_{aA}; (c) Z_p; (d) the transmission efficiency. [4544 $\angle 0.175$ rms, 17.924 $\angle -14.036$ A, 98.71%.

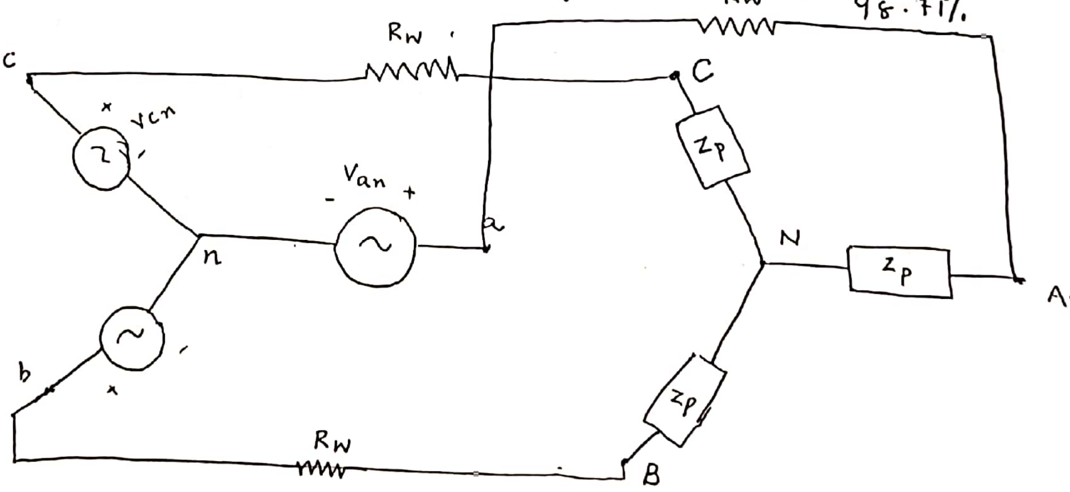


Fig P7.13

Phasor diagrams

Since ~~the~~ phasors are complex numbers, they may be represented by vectors in a plane, where operations such as addition of phasors may be carried out geometrically. Such a sketch is called phasor diagram and may be helpful in analyzing ac steady state circuits — i.e. phasor diagrams may be used to derive the solution geometrically.

The following rules should be observed when developing a phasor diagram.

1. Each voltage or current is represented by a phasor drawn on the complex Argand plane. The lengths of phasors are drawn proportional to the

magnitudes of voltages or currents they represent and their arguments must be ~~conte~~ consistent with the corresponding phase angle differences.

a) Phasors representing the voltage and the current of a resistor have the same argument i.e with the same orientation.

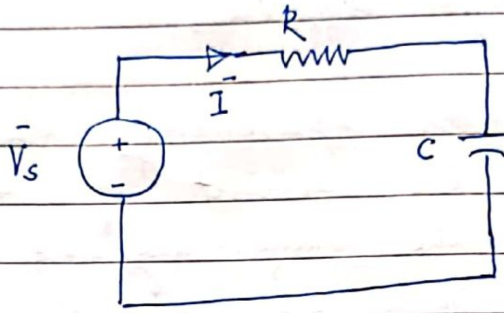
b) The phasor representing the current of a capacitor is drawn with an angle 90° leading the phasor of the same capacitor.

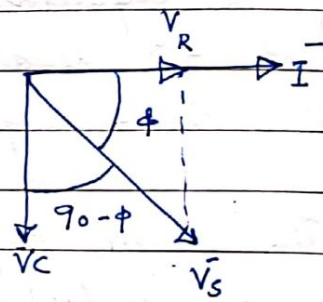
c) The phasor representing the current of an inductor is drawn with an angle 90° lagging the phasor of the voltage of the same inductor.

2. Summation, subtraction, multiplication and division are performed in accordance with the rules of elementary complex number manipulation.

a) Summation / subtraction of two phasors are performed vectorically according to the parallelogram law.

b) The product (quotient) of two phasors has magnitude equal to the algebraic product (quotient) of the individual magnitude and argument equal to the sum (difference) of the individual arguments.





$$\phi = \tan^{-1} \frac{V_c}{V_R}$$

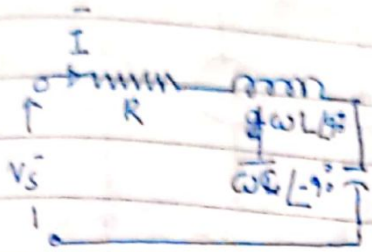
$$= \tan^{-1} \omega C R$$

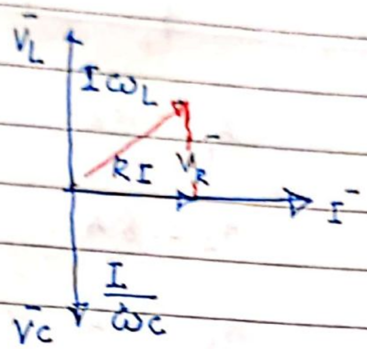
Let $\bar{V}_s(t) = V_{sm} \cos(\omega t)$

$$i(t) = I_m \cos(\omega t + \phi)$$

$$V_c = V_{cm} \cos(\omega t + 90 - \phi)$$

$$\bar{V}_R = V_{Rc} \cos(\omega t + \phi)$$





$$\dot{V}_S = \sqrt{(RI)^2 + I^2\left(\omega L - \frac{1}{\omega C}\right)^2}$$

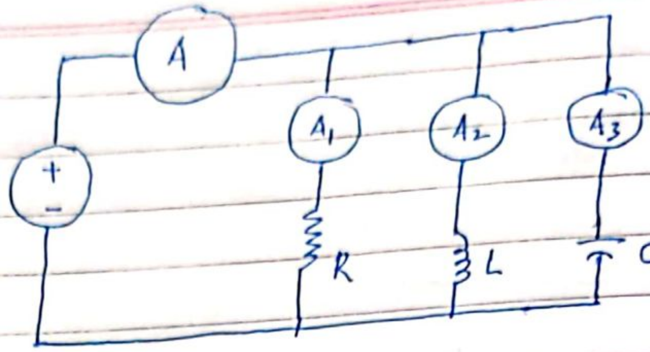
$$\dot{V}_S = I\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

$$V_S = I\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

$$I = \frac{V_S}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

$$\phi = \tan^{-1}\left(\frac{\omega L - \frac{1}{\omega C}}{R}\right)$$

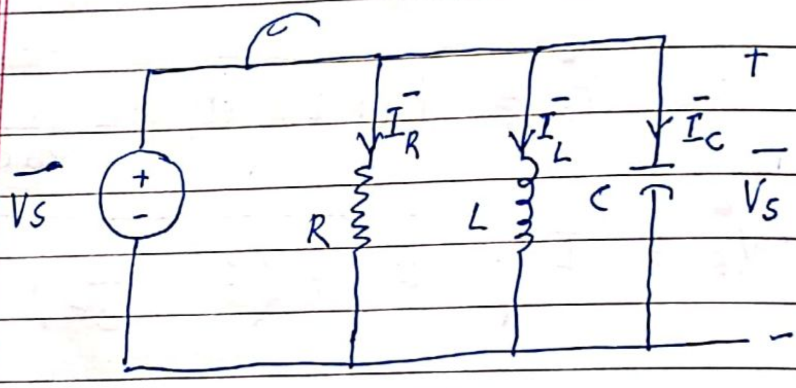
If $\bar{V}_S =$



$A_1 = 5A$

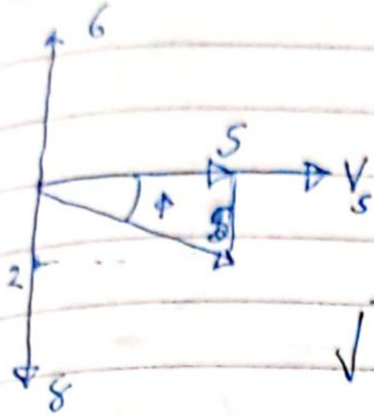
$A_2 = 8A$

$A_3 = 6A$

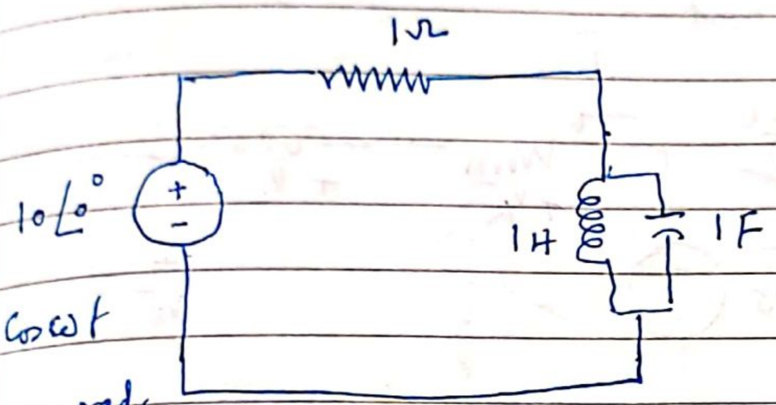


Since voltage is common to all the elements we will take it as our reference phasor i.e

$$\bar{V}_S = V_S \angle 0$$

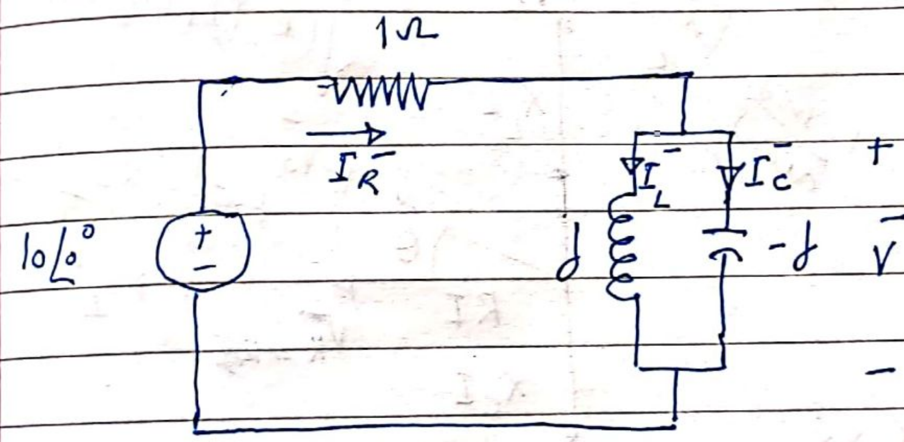


$$\sqrt{2^2+5^2} = \sqrt{4+25} = \sqrt{29}\ A$$

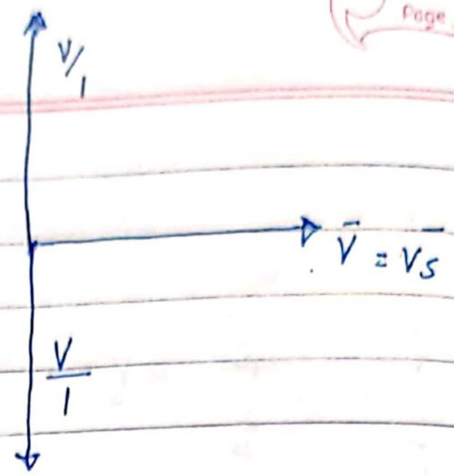


$\sqrt{2} \times 10 \cos \omega t$

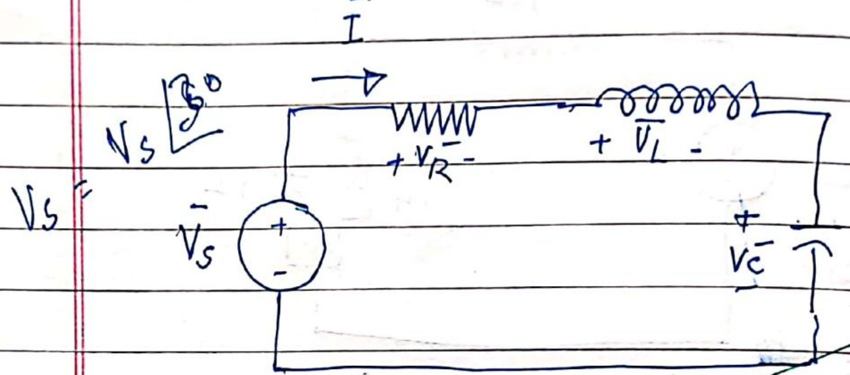
$\omega = 1\ rad/s$

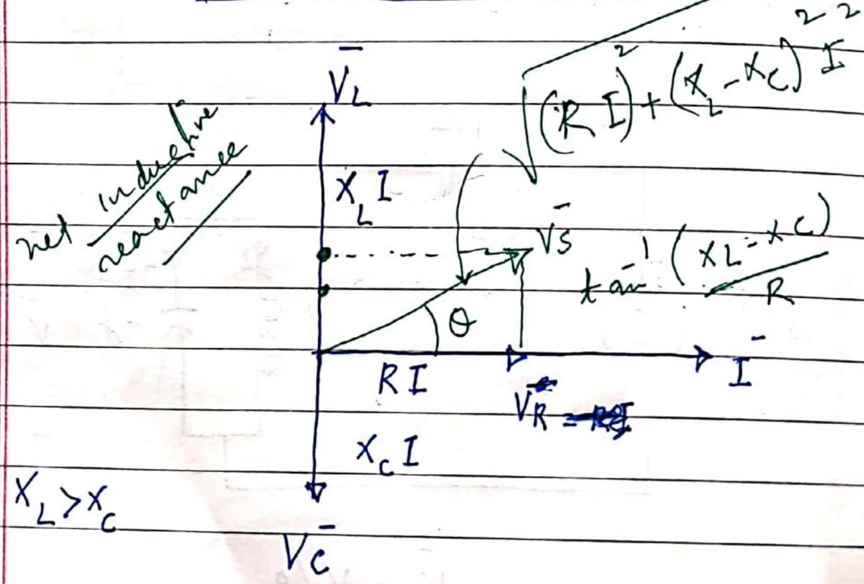


$$\bar{V} = V\angle 0$$



Phasor diagram for Series RLC circuit

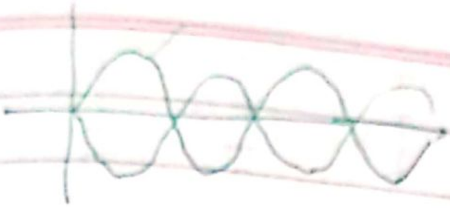




$\sqrt{(RI)^2 + (X_L - X_C)^2 I^2}$

$\tan^{-1}\left(\frac{X_L - X_C}{R}\right)$

Current lags the voltage by θ.



$X_C > X_L$

$\bar{V_S} = V_S \angle \phi$

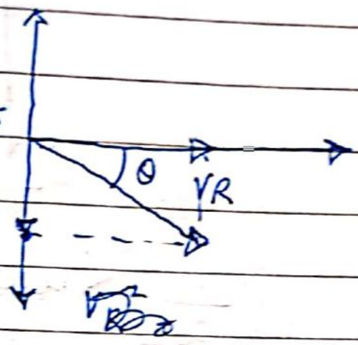
$\bar{I} =$

$\omega L > \frac{1}{\omega C}$

current i lags voltage Vs

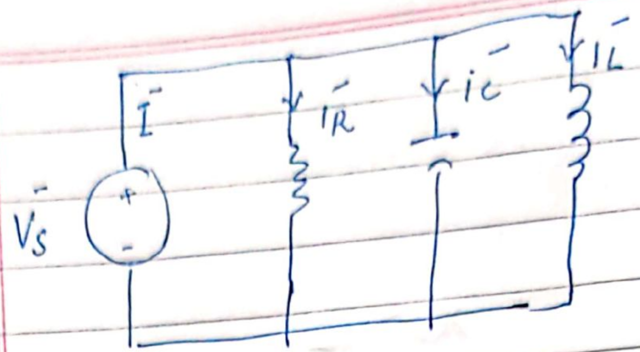
$\omega L < \frac{1}{\omega C}$

current i leads voltage

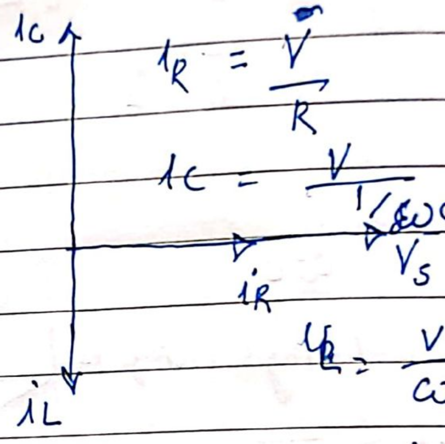


$\omega L = \frac{1}{\omega C}$ $\quad X_C = X_C$

current i in phase with voltage



$$I = \left| \frac{\bar{V}}{Z} \right|$$



$$i_R = \frac{\bar{V}}{R}$$

$$i_C = \frac{V}{1/\omega C}$$

$$i_L = \frac{V}{\omega L}$$

$X_C < X_L$

$\frac{1}{\omega L} < \omega C$ Vab lags i

$\omega L > \frac{1}{\omega C}$ i leads $\bar{V}$

$\omega L < \frac{1}{\omega C}$ i lags $\bar{V}$

$\omega L = \frac{1}{\omega C}$ i in phase with V