Nodal Analysis

Nodal analysis uses node voltages as circuit variables rather than currents as used in mesh analysis. This is very convenient as well reduces the number of equations.

Nodal analysis is very comfortable to be used for circuits that have only current sources where as mesh analysis is comfortable to use used with circuit having a voltage sources.

Steps involved in nodal analysis are

Step I Select one node as refrence node. Assign node voltages V_1, V_2, V_3 ..... to rest of nodes. These voltages are referenced w.r.t. reference voltage.

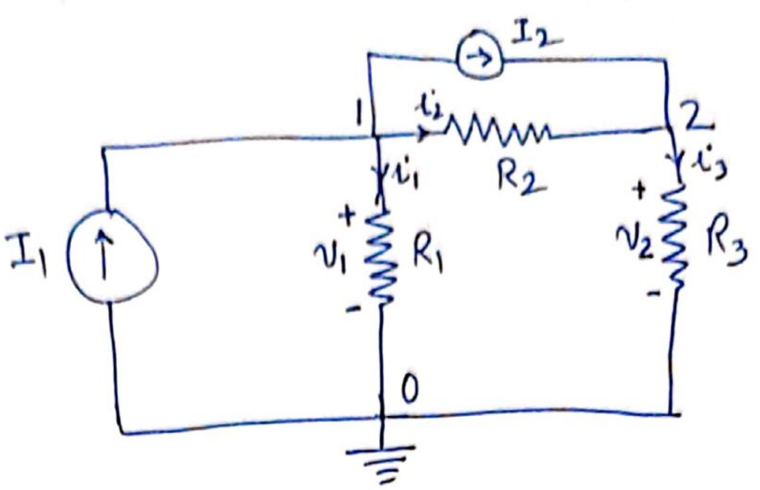
Step II Apply KCL at nodes V_1, V_2, V_3 ..... using ohms law to express branch currents.

Step III Solve the simulataneous equations to obtain unknown node voltages. Subsequently find branch currents through all elements.

The refrence node is commonly called ground and it is assumed to have zero potential.

Symbol →

Consider a circuit given below:-



Node 0 $\longrightarrow$ reference.

There are two other nodes ① & ② whose voltages w.r.t. to reference are selected as V_1 & V_2.

Apply KCL at ① & ②.

$$I_1 = I_2 + i_1 + i_2 \quad \& \quad I_2 + i_2 = i_3$$

now $i_1 = \dfrac{V_1 - 0}{R_1}$; $\quad i_2 = \dfrac{V_1 - V_2}{R_2}$, $\quad i_3 = \dfrac{V_2 - 0}{R_3}$;

$$I_1 = I_2 + \frac{V_1}{R_1} + \frac{V_1 - V_2}{R_2} \quad \text{——— ①.}$$

or $$I_2 = \frac{V_2 - 0}{R_3} - \frac{V_1 - V_2}{R_2} \quad \text{——— ②.}$$

$$\left(\frac{V_1}{R_1} + \frac{V_1}{R_2}\right) - \frac{V_2}{R_2} = I_1 - I_2 \quad \text{——— ③.}$$

$$\left(\frac{V_2}{R_3} + \frac{V_2}{R_2}\right) - \frac{V_1}{R_2} = I_2 \quad \text{——— ④.}$$

$$\left(\frac{1}{R_1} + \frac{1}{R_2}\right) V_1 - \left(\frac{1}{R_2}\right) V_2 = I_1 - I_2$$

$$\left(\frac{1}{R_2} + \frac{1}{R_3}\right) V_2 - \left(\frac{1}{R_2}\right) V_1 = I_2 \, ;$$

$$\left(\frac{1}{R_1}+\frac{1}{R_2}\right)V_1 - \left(\frac{1}{R_2}\right)V_2 = I_1 - I_2$$

$$\left(-\frac{1}{R_2}\right)V_1 + \left(\frac{1}{R_2}+\frac{1}{R_3}\right)V_2 = I_2 .$$

we can express in matrix form as;

$$\begin{bmatrix} \left(\frac{1}{R_1}+\frac{1}{R_2}\right) & -\frac{1}{R_2} \\ -1/R_2 & \left(\frac{1}{R_2}+\frac{1}{R_3}\right) \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} I_1 - I_2 \\ I_2 \end{bmatrix}$$

We can solve these two equations to find V_1 & V_2.

How to write the above equations by mere inspection;

The first element is (2×2) matrix;

All diagonal elements are +ve and off diagonal elements are negative. The first element is sum of reciprocals of resistances connected to node 1. The second element is minus times reciprocal of resistance b/w node 1 & node 2. The third similarly -ve of ~~sum~~ reciprocal of resistance b/w 2 & 1 and the last is sum of reciprocal of resistances connected to nodes ②.

The 2nd element is matrix of node voltages of V_1 & V_2.

The third element is ~~sum~~ matrix of the currents; first element Sum of currents entering and leaving the node. Current entering is taken +ve and current away from node is -ve..

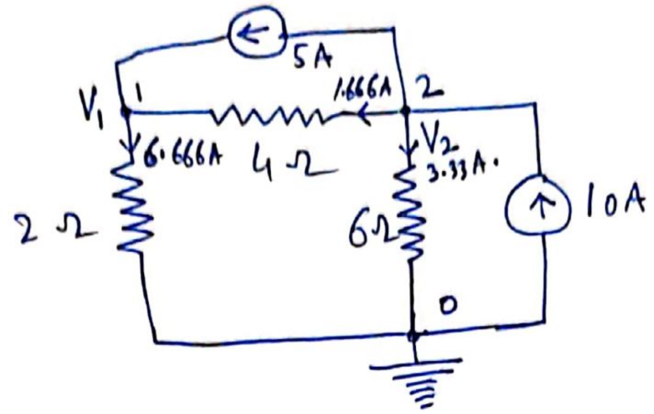
$\frac{1}{4}+\frac{1}{6}$

$\frac{3+2}{12} = \frac{5}{12}$

$12\overline{)50}.417$

48, 20, 12, 80

Example :-



Two nodes ① & ② with V_1 & V_2 . and node 0 as refrence nodes.

$$\begin{bmatrix} \left(\frac{1}{2}+\frac{1}{4}\right) & -\frac{1}{4} \\ -\frac{1}{4} & \left(\frac{1}{4}+\frac{1}{6}\right) \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 5 \\ 10-5 \end{bmatrix}$$

$$\begin{bmatrix} 0.75 & -0.25 \\ -0.25 & 0.41 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 5 \\ 5 \end{bmatrix}$$

$$0.75\,V_1 - 0.25\,V_2 = 5$$

$$-0.25\,V_1 + 0.41\,V_2 = 5$$

solve for V_1 & V_2 to ~~get~~ ; ;

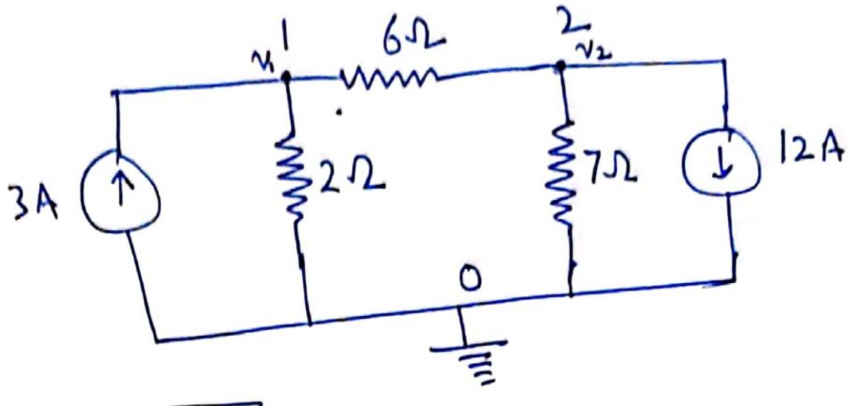
$$\boxed{\begin{aligned} V_1 &= 13.333\,V \\ V_2 &= 20\,V. \end{aligned}}$$

current through 6 Ω = $\frac{20}{6} = 3.33\,A.$

current through 4 Ω = $\frac{V_2 - V_1}{4} = \frac{20-13.33}{4} = \frac{6.667}{4} = \frac{6.67}{4} = 1.66A.$

Current through 2 Ω = $\frac{V_1}{2} = \frac{13.333}{2} = 6.666\,A.$

Example



Find V_1 & V_2

Write

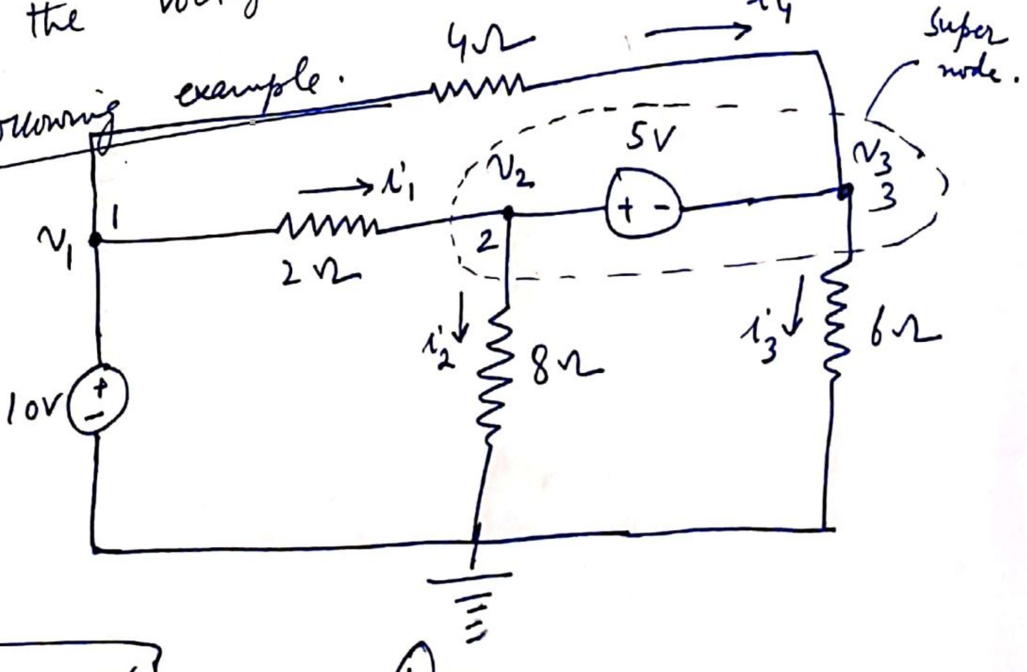
$\boxed{V_1 = -6V}$; $\boxed{V_2 = -42V}$

Nodal Analysis with voltage sources.

We consider two possibilities;

Case 1 If a voltage source is connected between the refrence node and a non refrence node. we simply set the voltage at the non- refrence node equal to voltage of the voltage source.

Consider following example.



$\therefore$ $\boxed{v_1 = 10V}$ ——— (1).

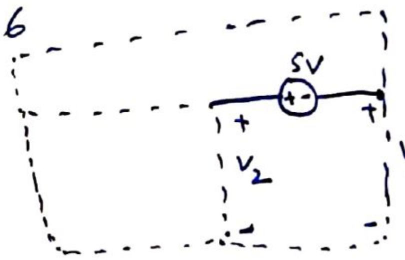
Case 2 If the voltage source (Dependant or independant) is connected b/w two non refrence nodes, the two non refrence nodes form a generalised node or super node. we apply KCL & KVL to determine the node voltages.

Thus nodes (2) & (3) form a super node.

KCL at super node :,

$$i_1' + i_4' = i_2' + i_3'$$

$$\frac{V_1 - V_2}{2} + \frac{V_1 - V_3}{4} = \frac{V_2 - 0}{8} + \frac{V_3 - 0}{6} \quad \text{——— (2)}.$$



KVL in super node :-

$$V_2 - 5 - V_3 = 0$$

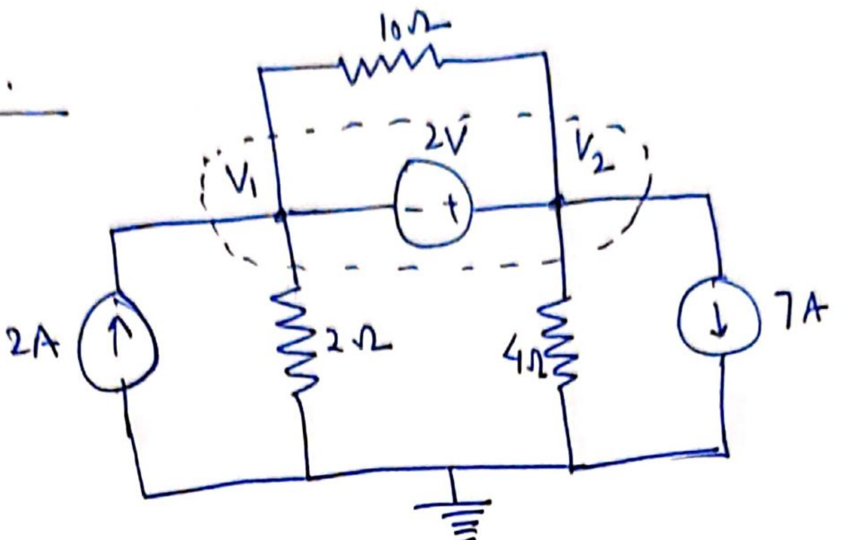
$$\boxed{V_2 - V_3 = 5} \quad \text{——— (3)}.$$

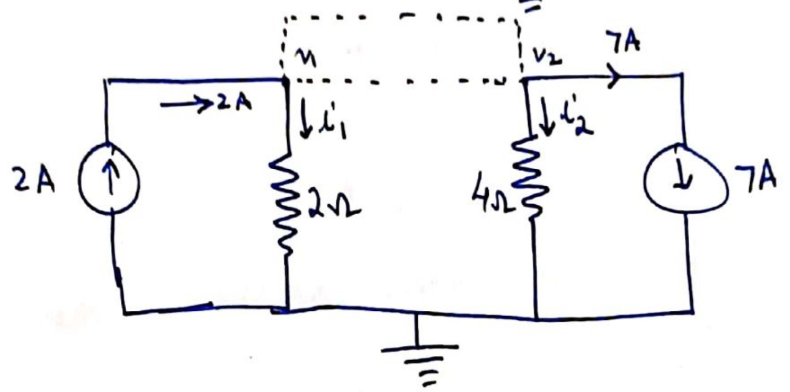
Solve Eq (1), (2), & (3). to get V_1, V_2 & V_3.

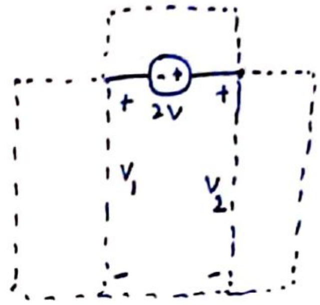
Properties of a Super node.

1. The voltage source inside a supernode provides a constraint equation needed to solve for node voltages.
2. A super node has no voltage of its own
3. A super node requires the application of both KCL and KVL.

Example.







Apply KCL $\qquad 2 = i_1 + i_2 + 7$

$$2 = \frac{V_1}{2} + \frac{V_2}{4} + 7$$

$$\frac{V_1}{2} + \frac{V_2}{4} = -5$$

$$\frac{2V_1 + V_2}{4} = -5 \qquad \text{or } 2V_1 + V_2 = -20$$

$$\boxed{V_2 = -20 - 2V_1}$$

Apply KVL ; $\qquad V_1 + 2 - V_2 = 0.$

$$V_1 - V_2 = -2 \qquad \therefore V_1 = V_2 - 2;$$

$$\therefore V_2 = -20 - 2(V_2 - 2)$$

$$= -20 - 2V_2 + 4$$

$$V_2 + 2V_2 = -16$$

$$3V_2 = -16$$

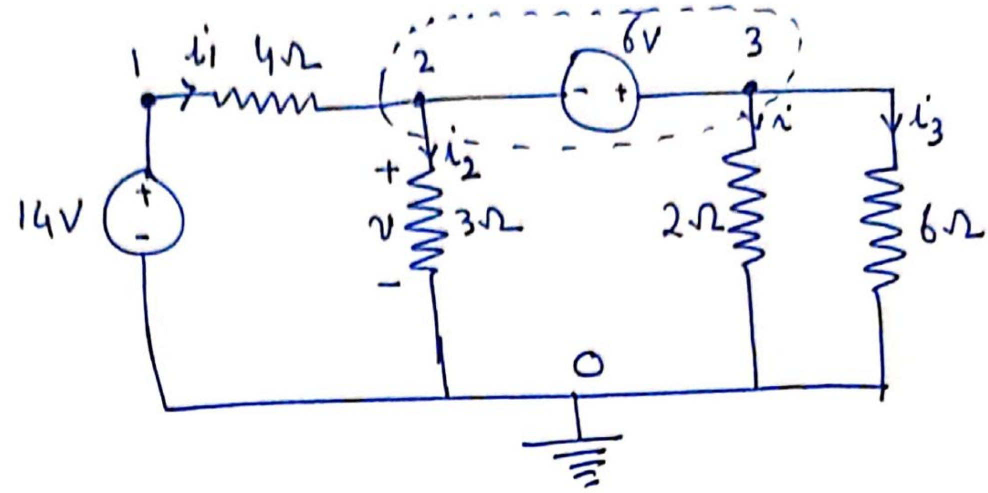
$$\boxed{V_2 = -16/3 = -5.333}$$

$$\therefore V_1 = -5.333 - 2$$

$$\boxed{V_1 = -7.333}$$

$$\boxed{\begin{aligned} V_1 &= -7.333\,V \\ V_2 &= -5.333\,V \end{aligned}}$$ Ans.

Example.

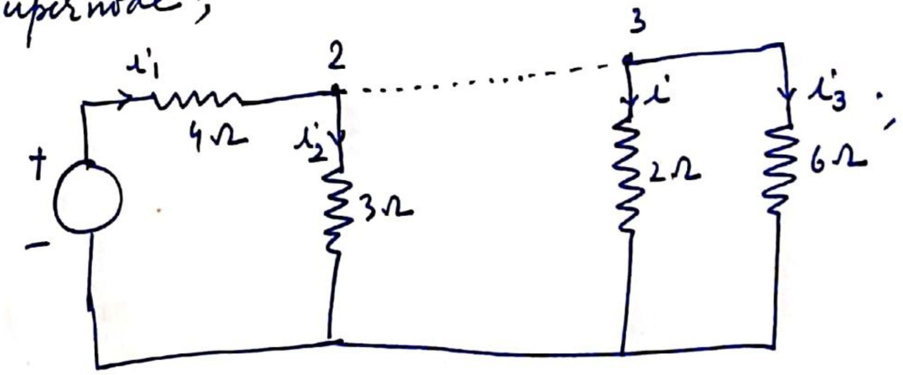


Sol; voltage source between 0 & 1 ; ∴ 14V;

$$\boxed{V_1 = 14V}$$

~~Note~~ voltage source b/w 2 & 3 ; 6V;
non-refrence nodes;

resuets a supernode;



KCL at Supernode.

$$i_1 = i_2 + i + i_3$$

$$\frac{V_1 - V_2}{4} = \frac{V_2}{3} + \frac{V_3}{2} + \frac{V_3}{6}$$

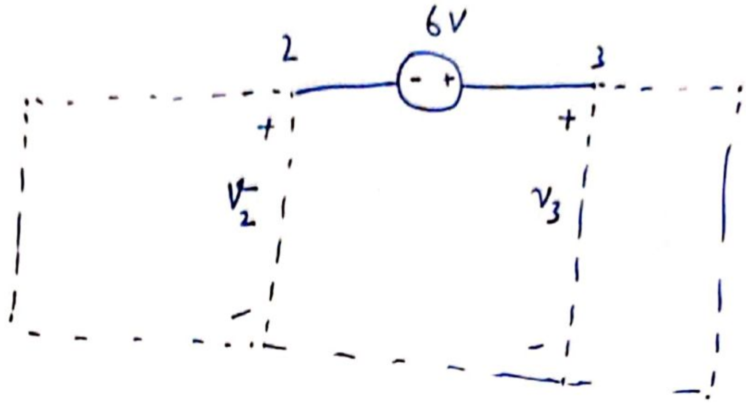
$$\frac{V_1}{4} = \frac{V_2}{4} + \frac{V_2}{3} + \frac{V_3}{2} + \frac{V_3}{6}$$

$$= \frac{3V_2 + 4V_2 + 6V_3 + 2V_3}{12}$$

$$\frac{V_1}{4} = \frac{7V_2 + 8V_3}{\cancel{12}\,3}$$

$$\boxed{V_1 = \frac{7V_2 + 8V_3}{3}}$$

KVL in supernode



$V_2 + 6 - V_3 = 0.$

$V_3 = V_2 + 6$; Also $V_1 = 14.$

$\therefore \quad 14 = \frac{7V_2 + 8(V_2+6)}{3}$

$7V_2 + 8V_2 + 48 = 42$

$15V_2 = -6$

$V_2 = -\frac{\cancel{6}\,2}{\cancel{15}\,5} = \quad V_2 = \frac{-2}{5} = -0.4V$

$V_2 = 0.4 \times 1000 \text{ mV}$

$\boxed{V_2 = 400mV}$

$\therefore V_3 = -0.4V + 6$

$V_3 = 5.6V.$

$\therefore \quad i = \frac{5.6}{2} = 2.8A.$

$\boxed{i_1 = 2.8A}$