Lecture 16

Resistance Measurement

From the point of view of measurement, resistances are classified as :-

(A) Low Resistances :- of the order of 1 Ω and under e.g. series winding resistances, ammeter shunts, Cable lengths, contacts, etc.

(B) Medium Resistances : From 1 Ω to about 100 KΩ. Majority of the pieces of electrical apparatus used have resistances in this category.

(C) High Resistances :- From 100 KΩ onwards. e.g. insulation resistances, etc.

The above classification is not rigid but forms a guide as to the method of measurement to be adopted in any particular case.

(A) Measurement of low Resistance

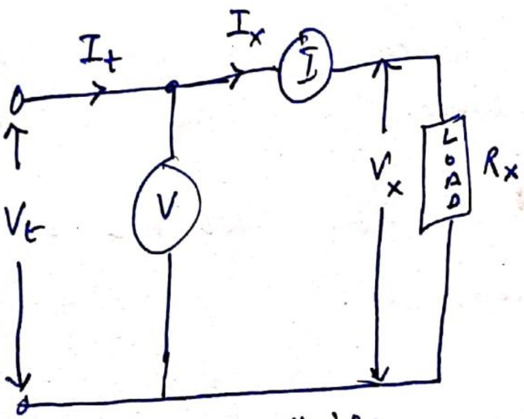
(1) Voltmeter - Ammeter Method The instruments required in this method are easily available in most laboratories. If the voltage V across resistor and the current I through the resistor are measured, the unknown resistance, R_x can

be calculated by ohm's law;

$$R_x = \frac{V}{I}$$

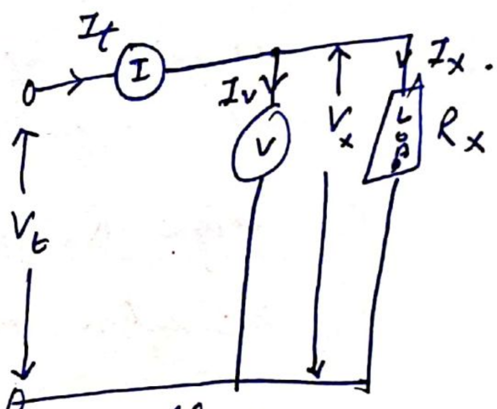
The equation shows that the conditions are not disturbed when the ammeter resistance is zero and voltmeter resistance is infinite.

Two configurations are possible;



(High)

Fig (a).



(low)

Fig (b).

For figure (a), the current supplied to the load is measured by ammeter, but the voltmeter measures the supply voltage rauther than the actual load voltage. To find the true voltage across load, the voltage drop across the ammeter must be substracted from the voltmeter reading.

Howener in fig (b), if the voltmeter is placed directly across the ~~load~~ resistor, it measures the true load voltage, but the ammeter is in error, by the amount of current drawn by voltmeter.

Let R_V = voltmeter resistance

Current through voltmeter $I_V = \frac{V}{R_V}$

Measured value of resistance, $R_m = \frac{V}{I}$;

Also $R_m = \frac{V}{I_V + I_x} = \frac{V}{\frac{V}{R_V} + \frac{V}{R_x}}$

$$R_m = \frac{V}{V\left(\frac{1}{R_V} + \frac{1}{R_x}\right)} = \frac{R_x R_V}{R_x + R_V}$$

$$R_x R_V = R_m R_x + R_m R_V$$

$$R_x (R_V - R_m) = R_m R_V$$

$$\boxed{R_x = \frac{R_m R_V}{R_V - R_m}} = \frac{R_m}{1 - R_m/R_V}$$

If voltmeter resistance is very very large ($\to \infty$),

$$\boxed{R_x = R_m}$$

In either configuration, an error is introduced in the measurement of R_x. The proper method of evaluation of R_x depends on the value of R_x and resistances of ammeter and voltmeter.

Fig (a) is suitable for measurement of high resistance values. The ammeter reads the true load current I_x. If R_x is very large as compared ammeter resistance, the error introduced by neglecting the voltage drop across the ammeter is negligible and V_t is very close to true load voltage V_x.

In Fig (b), the voltmeter reads the true load voltage, V_x and ammeter reads the supply current I_t. If R_x is smaller compared to internal resistance of voltmeter, the current drawn by voltmeter does not appreciably effect the total supply current and I_t is very close to true value of load current. There fig(b) is suitable for low resistance measurement.

Attention to fig (b).

For point C (Position 3).

$$P/Q = \frac{R + r_1}{S + r_2}$$

Consider point C choosen in such a manner that the lead resistance r is divided in the same ratio as P/Q; i.e. $\frac{r_1}{r_2} = P/Q$.

$$1 + \frac{r_1}{r_2} = 1 + P/Q$$

$$\frac{r_1 + r_2}{r_2} = \frac{P+Q}{Q}$$

$$\frac{r}{r_2} = \frac{P+Q}{Q}$$

$$\boxed{r_2 = \frac{Q}{P+Q} \cdot r}$$

$$\frac{r_2}{r_1} = \frac{Q}{P}$$

$$1 + \frac{r_2}{r_1} = 1 + \frac{Q}{P}$$

$$\frac{r_1 + r_2}{r_1} = \frac{P+Q}{P}$$

$$\frac{r}{r_1} = \left(\frac{P+Q}{P}\right)$$

$$\boxed{r_1 = \frac{P}{P+Q} \cdot r}$$

From above,

$$P/Q = \frac{R + \frac{Pr}{P+Q}}{S + \frac{Qr}{P+Q}}$$

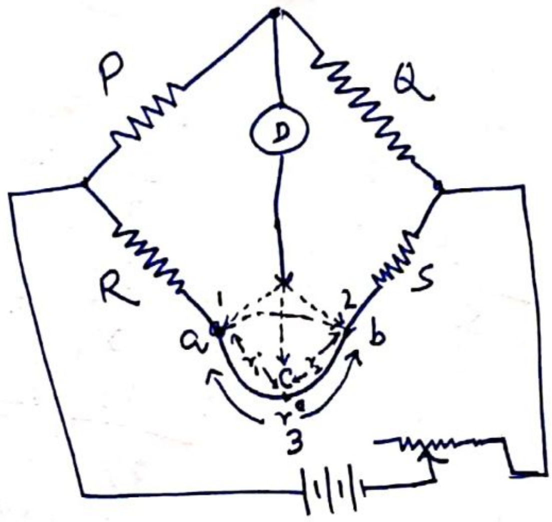
$$P/Q \cdot S + \frac{Pr}{P+Q} = R + \frac{Pr}{P+Q}$$

$$\boxed{R = P/Q \cdot S}$$

Thus there is no lead resistance in the final result.

(2) Kelvins Double Bridge Method

Kelvins Bridge is used for measurement of low resistances which are smaller than lead resistance. First consider a single Kelvins bridge, as shown below :—



The lead ab connecting R & S arms of bridge has a resistance which is comparable to resistance R (low resistance under measurement).

For switch position 1. (Point a)

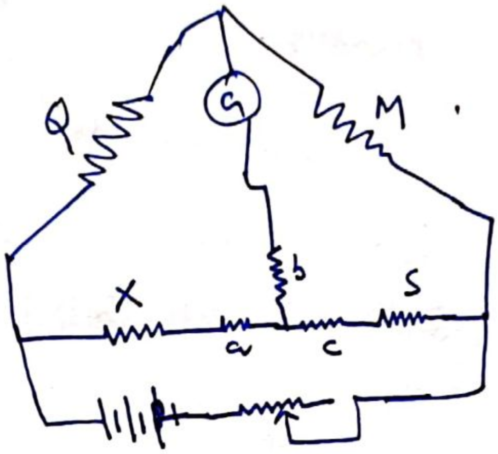
$$\frac{P}{Q} = \frac{R}{S + r} \Rightarrow R = \frac{P}{Q}(S + r).$$

$$\boxed{R = \frac{P}{Q} \cdot S + \frac{P}{Q} \cdot r}$$

for switch position 2. (Point b).

$$\frac{P}{Q} = \frac{R + r}{S} \Rightarrow \boxed{R = \frac{P}{Q} \cdot S - r}$$

Theory. Transform the Δ arrangement of q, m and r resistances into equivalent Y. the resultant circuit looks like ;



where $a = \frac{qr}{q+r+m}$; $b = \frac{qm}{q+r+m}$, $c = \frac{mr}{q+r+m}$;

At balance no current flows through the galvanometer ; $\therefore \frac{Q}{M} = \frac{X+a}{S+C}$

$$\frac{Q}{M}(S+C) = X + a$$

$$X = \frac{Q}{M}\cdot S + \frac{Q}{M}\cdot c - a$$

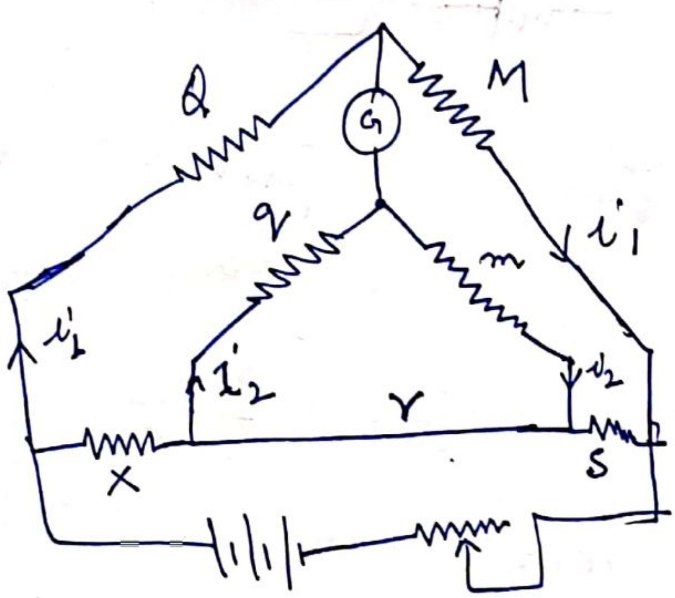
$$= \frac{Q}{M}\cdot S + \frac{Q}{M}\cdot\frac{mr}{q+r+m} - \frac{qr}{q+r+m}$$ → zero.

$$= \frac{Q}{M}\cdot S + \frac{mr}{q+r+m}\left[\frac{Q}{M} - \frac{q}{m}\right]$$

$$\therefore \boxed{X = \frac{Q}{M}\cdot S}$$

To practically find this point C corresponding to switch position 3 is very difficult. Thus Kelvins Single Bridge is very difficult to implement practically.

Thus Kelvins Double Bridge is proposed, which uses two pair of arms of same ratio as shown in following figure.



X = unknown low resistance

S = standard resistance of the order of same magnitude

r = lead resistance;

The basic condition to be met in KDB is

so that $\frac{X}{S} = \frac{Q}{M} = \frac{q}{m}$

$$\therefore \boxed{\frac{X}{S} = \frac{Q}{M} \cdot S}$$

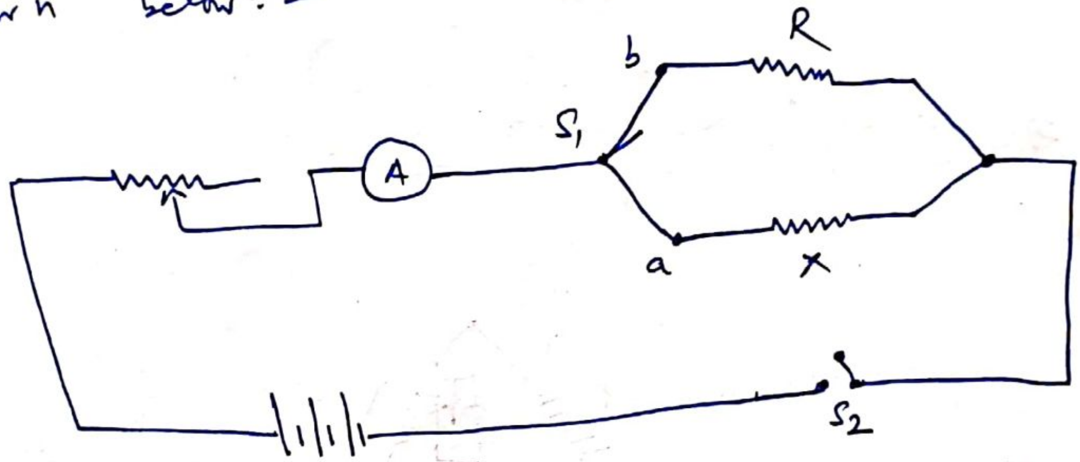
(B) Measurement of Medium Resistance

① Ammeter – Voltmeter Method

as explained already →

② Substitution Method :-

The circuit diagram for the method is shown below :-



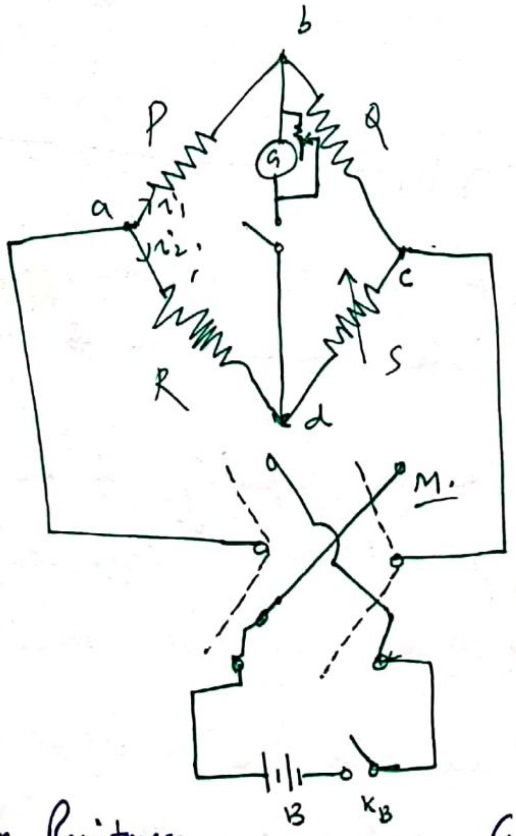
X = resistance to be measured ;
R = variable standard resistor ;

A battery of ample capacity is used for supply, since it is important in this experiment that the supply voltage shall be constant.

With S_2 closed and S_1 on stud a, the deflection of ammeter is observed. S_1 is thrown on stud "b" and the standard variable resistance R, adjusted untill the same deflection is obtained on the indicating instrument. The value of R which gives same deflection is the resistance of unknown X.

R & X should be large as compared with other resistances of the circuit. The accuracy depends upon the constancy of supply voltage, the resistance of circuit excluding X & R and upon sensitivity of indicating instrument and upon accuracy with which R is known.

3) Wheatstone's Bridge



P, Q are two known Resistances

S → known variable resistance

R = unknown resistance.

G = Galvanometer.

At balance the shorting is made zero giving free sensitivity of galvanometer.

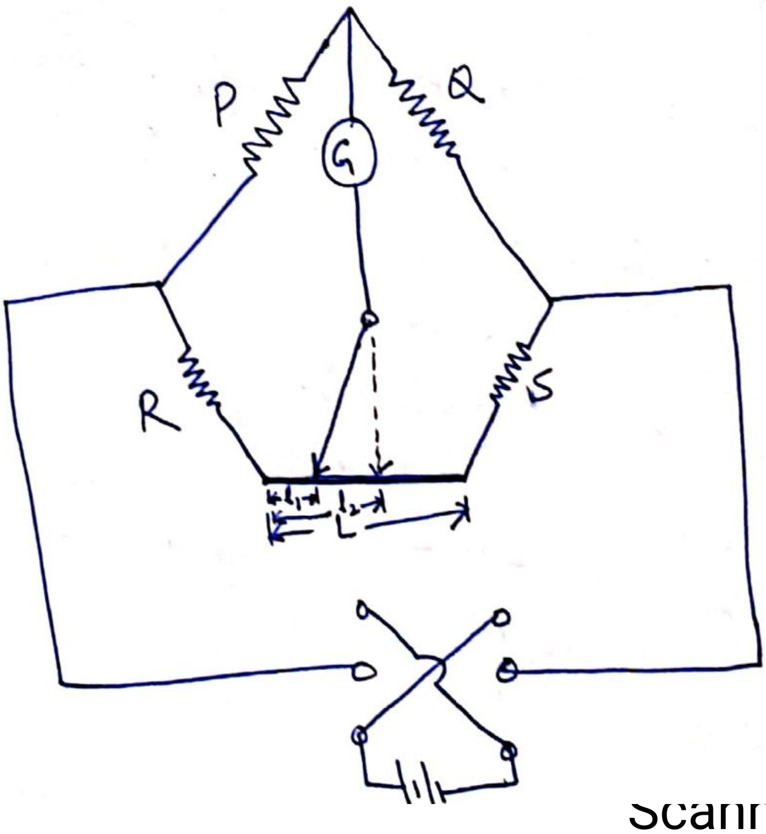
M = Reversing switch so that the current is made to flow in both directions and two seperate measurements of unknown resistance is made to eliminate thermo-electric errors.

At balance, obtained by adjusting S, same current flows through P & Q as well as R & S.

$$i_1 P = i_2 R$$
$$i_1 Q = i_2 S$$

$$P/Q = R/S \Rightarrow \boxed{R = \frac{P}{Q} \cdot S}$$

(4) Carey-Foster slide wire Bridge



under balanced Conditions, we have,

$$P/Q = \frac{R + l_1 r}{S + (L - l_1) r} \quad \text{--- (1)}$$

where r = resistance/length of slide wire, connecting R & S resistances.

now Interchange R & S and repeat the experiment, let us now say that bridge is balanced at l_2 length.

$$P/Q = \frac{S + l_2 r}{R + (L - l_2) r} \quad \text{--- (2)};$$

from eqn (1),

$$P/Q + 1 = \frac{R + l_1 r}{S + (L - l_1) r} + 1$$

$$= \frac{R + l_1 r + S + Lr - l_1 r}{S + (L - l_1) r}$$

$$P/Q + 1 = \frac{R + S + Lr}{S + (L - l_1) r} \quad \text{--- (3)}.$$

from eqn (2), we have,

$$P/Q + 1 = \frac{S + l_2 r}{R + (L - l_2) r} + 1$$

$$= \frac{S + l_2 r + R + Lr - l_2 r}{R + (L - l_2) r}.$$

$$P/Q + 1 = \frac{S + R + Lr}{R + (L - l_2) r}. \quad \text{--- (4)}.$$

Equating (3) & (4),

$$\frac{R + S + Lr}{S + (L - l_1) r} = \frac{R + S + Lr}{R + (L - l_2) r};$$

$$S + (L - l_1) r = R + (L - l_2) r.$$

$$S + Lr - l_1 r = R + Lr - l_2 r$$

$$S - R = (l_1 - l_2) r \quad \text{--- (5)}.$$

Let us now shunt the resistance S with a known resistance and let the equivalent resistance is S′; let now lengths l_1' & l_2' lead to balance;

Repeat the whole experiment;

$$S' - R = (l_1' - l_2')\,r \quad \text{——— (6).}$$

from (5) & (6), we get;

$$\frac{S-R}{l_1 - l_2} = \frac{S'-R}{l_1' - l_2'}$$

$$(S-R)(l_1'-l_2') = (S'-R)(l_1-l_2).$$

$$\Rightarrow \quad (S-R)(l_1'-l_2') = S'(l_1-l_2) - R(l_1-l_2).$$

$$S(l_1'-l_2') - R(l_1'-l_2') = S'(l_1-l_2) - R(l_1-l_2)$$

$$S(l_1'-l_2') - S'(l_1-l_2) = R(l_1'-l_2') - R(l_1-l_2)$$

$$= R\left[(l_1'-l_2') - (l_1-l_2)\right].$$

$$\therefore R = \frac{S(l_1'-l_2') - S'(l_1-l_2)}{(l_1'-l_2') - (l_1-l_2)};$$

The whole experiment can be repeated by interchanging the battery connections and let now value obtained is R′. The average of R & R′ will lead to final result.

Measurement of High Resistance

The Common examples of high resistance are:-

i) Insulation resistance of components, machines, cables, etc.

ii) Resistance of Vacuum Tubes.

iii) Leakage resistance of Capacitors

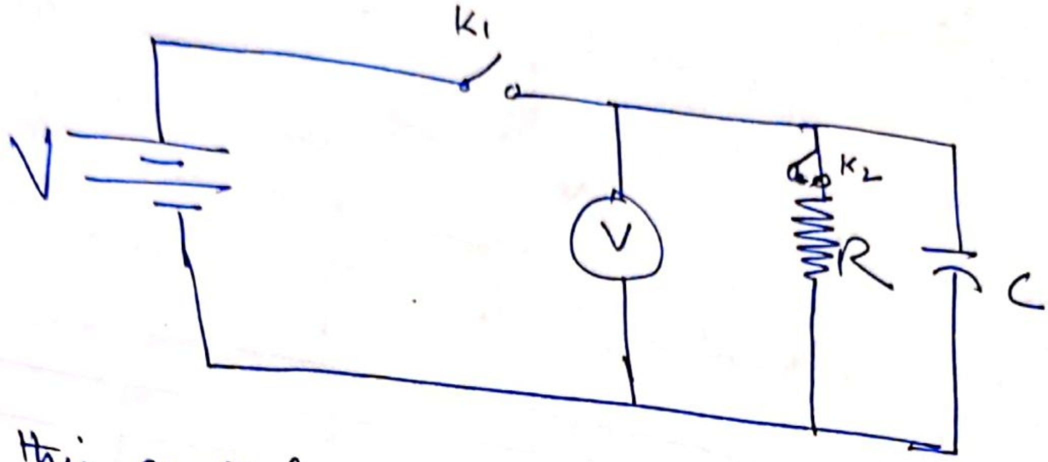
(iv) Resistance b/w two faces of unit area seperated by unit distance. (volume resistivity of material).

(v) Surface resistivity i.e. resistance b/w two lines of unit length and unit distance apart.

The different methods employed are

1. Direct Deflection method
2. Loss of Charge method
4. Megger.
3) Megohm Bridge.

(2) Loss of charge method



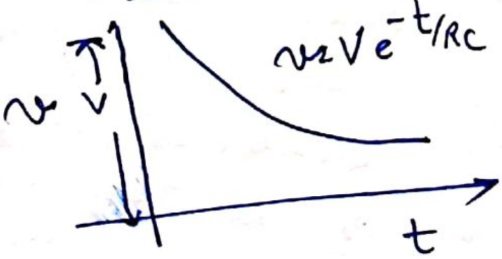
In this method, the resistance R to be measured is connected in parallel with a Capacitor C and an electrostatic voltmeter. Initially the Capacitor is charged to some suitable voltage by means of a battery having voltage V. This is done by closing switch K_1 and opening switch K_2. The Capacitor is then discharged through R by closing K_2 and opening K_1. The terminal voltage is observed over a Considerable period of time during discharge.

The voltage across the capacitor at any instant t, after the application of voltage is:

$$v = Ve^{-t/RC} \quad \text{or} \quad \frac{V}{v} = e^{t/RC},$$

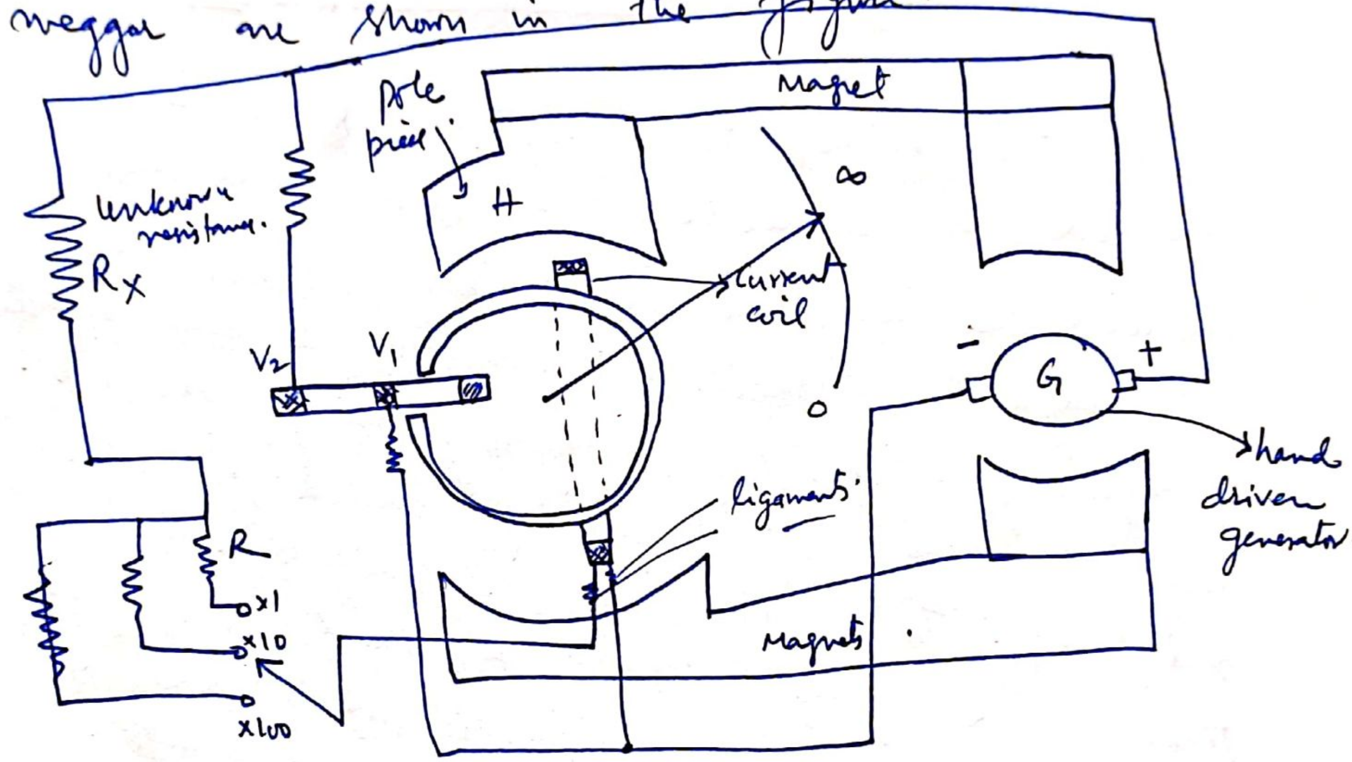
Insulation resistance,

$$R = \frac{t}{C \log_e V/v} = \frac{0.4343\,t}{C \log_{10} \frac{V}{v}}$$



3) Meggar :-

The principle of ratio ohmmeter is adopted to application in portable instruments measuring insulation resistance. This principle forms the basis of insulation testing instrument called Meggar. Meggar is also called insulation Tester. The essential parts of a meggar are shown in the figure.



The current coil is similar to that of a PMMC instrument. There are two voltage coils V_1 & V_2. The voltage coil V_1 embraces (threads over), the annular magnetic core. The voltage coil V_1 is in a weak magnetic field when the pointer is at "∞" position and hence this coil can exert very little torque.

The torque exerted by the voltage coil increases as it moves into a stronger field and this torque is maximum when it is under the pole face. and under this condition, the pointer is at its zero end of the resistance scale.

In order to modify further the torque in the voltage circuit, another voltage coil V_2 is used. This coil is also so located that it moves into stronger field as the pointer moves from the infinite position towards the zero position of the resistance cable. The coil finally embraces (threads around), the extension H of the pole piece. The combined action of two voltage coils V_1 & V_2 may be considered as though the coils constituted a spring of variable stiffness; being very stiff near the zero end of scale where the current in the current coil is very large (on a/c of unknown resistance R_x being small) and very weak near the ∞ end of the scale where the current in the current coil is very small on a/c of unknown resistance R_x being very large.

Thus this effect compresses the low resistance portion of the scale and opens up the high resistance portion of the scale. This is a great advantage since this instrument is meant to be used as insulation tester and the insulation resistances are quite high.

The voltage range of the instrument can be controlled by a voltage selector switch. This can be done by varying resistance 'R' connected in series with the current coil. The test voltages usually are 500V, 1000V & 2500V. These are generated by a hand cranked generator G. A centrifugal clutch is incorporated in the generator drive mechanism which slips at a predetermined speed so that a constant voltage is applied to the insulation under test.