CHAPTER 11

SYMMETRICAL COMPONENTS AND SEQUENCE NETWORKS

One of the most powerful tools for dealing with unbalanced polyphase circuits is the *method of symmetrical components* introduced by C. L. Fortescue.[1] Fortescue's work proves that an unbalanced system of n related phasors can be resolved into n systems of balanced phasors called the *symmetrical components* of the original phasors. The n phasors of each set of components are equal in length, and the angles between adjacent phasors of the set are equal. Although the method is applicable to any unbalanced polyphase system, we confine our discussion to three-phase systems.

In a three-phase system which is normally balanced, unbalanced fault conditions generally cause unbalanced currents and voltages to exist in each of the phases. If the currents and voltages are related by constant impedances, the system is said to be *linear* and the principle of superposition applies. The voltage response of the linear system to the unbalanced currents can be determined by considering the separate responses of the individual elements to the symmetrical components of the currents. The system elements of interest are the machines, transformers, transmission lines, and loads connected to Δ or Y configurations.

[1] C. L. Fortescue, "Method of Symmetrical Coordinates Applied to the Solution of Polyphase Networks," *Transactions of AIEE*, vol. 37, 1918, pp. 1027–1140.

In this chapter we study symmetrical components and show that the response of each system element depends, in general, on its connections and the component of the current being considered. Equivalent circuits, called ***sequence circuits***, will be developed to reflect the separate responses of the elements to each current component. There are three equivalent circuits for each element of the three-phase system. By organizing the individual equivalent circuits into networks according to the interconnections of the elements, we arrive at the concept of three *sequence networks*. Solving the sequence networks for the fault conditions gives symmetrical current and voltage components which can be combined together to reflect the effects of the original unbalanced fault currents on the overall system.

Analysis by symmetrical components is a powerful tool which makes the calculation of unsymmetrical faults almost as easy as the calculation of three-phase faults. Unsymmetrical faults are studied in Chap. 12.

11.1 SYNTHESIS OF UNSYMMETRICAL PHASORS FROM THEIR SYMMETRICAL COMPONENTS

According to Fortescue's theorem, three unbalanced phasors of a three-phase system can be resolved into *three balanced systems* of phasors. The balanced sets of components are:

1. *Positive-sequence components* consisting of three phasors equal in magnitude, displaced from each other by 120° in phase, and having the same phase sequence as the original phasors,
2. *Negative-sequence components* consisting of three phasors equal in magnitude, displaced from each other by 120° in phase, and having the phase sequence opposite to that of the original phasors, and
3. *Zero-sequence components* consisting of three phasors equal in magnitude and with zero phase displacement from each other.

It is customary when solving a problem by symmetrical components to designate the three phases of the system as *a*, *b*, and *c* in such a manner that the phase sequence of the voltages and currents in the system is *abc*. Thus, the phase sequence of the positive-sequence components of the unbalanced phasors is *abc*, and the phase sequence of the negative-sequence components is *acb*. If the original phasors are voltages, they may be designated V_a, V_b, and V_c. The three sets of symmetrical components are designated by the additional superscript 1 for the positive-sequence components, 2 for the negative-sequence components, and 0 for the zero-sequence components. Superscripts are chosen so as not to confuse bus numbers with sequence indicators later on in this chapter. The positive-sequence components of V_a, V_b, and V_c are $V_a^{(1)}$, $V_b^{(1)}$, and $V_c^{(1)}$, respectively. Similarly, the negative-sequence components are $V_a^{(2)}$, $V_b^{(2)}$,

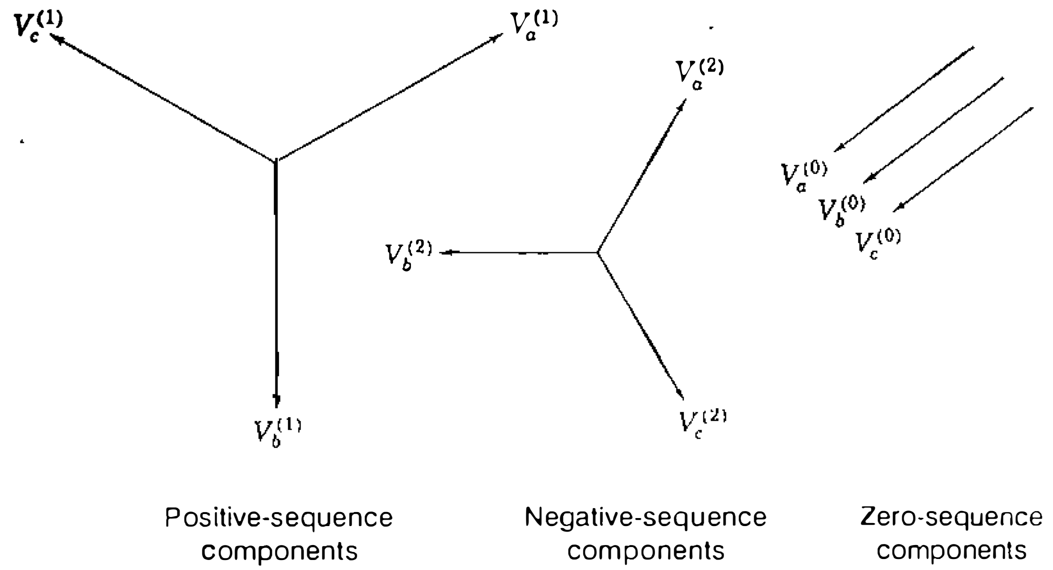


FIGURE 11.1
Three sets of balanced phasors which are the symmetrical components of three unbalanced phasors.

and $V_c^{(2)}$, and the zero-sequence components are $V_a^{(0)}$, $V_b^{(0)}$, and $V_c^{(0)}$, respectively. Figure 11.1 shows three such sets of symmetrical components. Phasors representing currents will be designated by I with superscripts as for voltages.

Since each of the original unbalanced phasors is the sum of its components, the original phasors expressed in terms of their components are

$$V_a = V_a^{(0)} + V_a^{(1)} + V_a^{(2)} \tag{11.1}$$

$$V_b = V_b^{(0)} + V_b^{(1)} + V_b^{(2)} \tag{11.2}$$

$$V_c = V_c^{(0)} + V_c^{(1)} + V_c^{(2)} \tag{11.3}$$

The synthesis of a set of three unbalanced phasors from the three sets of symmetrical components of Fig. 11.1 is shown in Fig. 11.2.

The many advantages of analysis of power systems by the method of symmetrical components will become apparent gradually as we apply the method to the study of unsymmetrical faults on otherwise symmetrical systems. It is sufficient to say here that the method consists in finding the symmetrical components of current at the fault. Then, the values of current and voltage at various points in the system can be found by means of the bus impedance matrix. The method is simple and leads to accurate predictions of system behavior.

11.2 THE SYMMETRICAL COMPONENTS OF UNSYMMETRICAL PHASORS

In Fig. 11.2 we observe the synthesis of three unsymmetrical phasors from three sets of symmetrical phasors. The synthesis is made in accordance with Eqs.

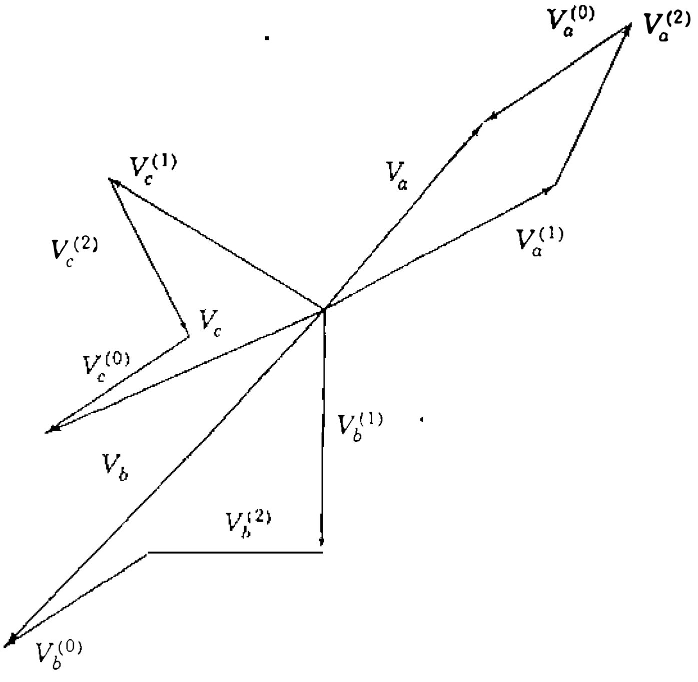


FIGURE 11.2
Graphical addition of the components shown in Fig. 11.1 to obtain three unbalanced phasors.

(11.1) through (11.3). Now let us examine these same equations to determine how to resolve three unsymmetrical phasors into their symmetrical components.

First, we note that the number of unknown quantities can be reduced by expressing each component of V_b and V_c as the product of a component of V_a and some function of the operator $a = 1\angle 120°$, which was introduced in Chap. 1. Reference to Fig. 11.1 verifies the following relations:

$$
\begin{aligned}
V_b^{(0)} &= V_a^{(0)} & V_c^{(0)} &= V_a^{(0)} \\
V_b^{(1)} &= a^2 V_a^{(1)} & V_c^{(1)} &= a V_a^{(1)} \\
V_b^{(2)} &= a V_a^{(2)} & V_c^{(2)} &= a^2 V_a^{(2)}
\end{aligned}
\tag{11.4}
$$

Repeating Eq. (11.1) and substituting Eqs. (11.4) in Eqs. (11.2) and (11.3) yield

$$V_a = V_a^{(0)} + V_a^{(1)} + V_a^{(2)} \tag{11.5}$$

$$V_b = V_a^{(0)} + a^2 V_a^{(1)} + a V_a^{(2)} \tag{11.6}$$

$$V_c = V_a^{(0)} + a V_a^{(1)} + a^2 V_a^{(2)} \tag{11.7}$$

or in matrix form

$$
\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} V_a^{(0)} \\ V_a^{(1)} \\ V_a^{(2)} \end{bmatrix} = \mathbf{A} \begin{bmatrix} V_a^{(0)} \\ V_a^{(1)} \\ V_a^{(2)} \end{bmatrix}
\tag{11.8}
$$

where, for convenience, we let

$$\mathbf{A} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \tag{11.9}$$

Then, as may be verified easily,

$$\mathbf{A}^{-1} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \tag{11.10}$$

and premultiplying both sides of Eq. (11.8) by $\mathbf{A}^{-1}$ yields

$$\begin{bmatrix} V_a^{(0)} \\ V_a^{(1)} \\ V_a^{(2)} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \mathbf{A}^{-1}\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} \tag{11.11}$$

which shows us how to resolve three unsymmetrical phasors into their symmetrical components. These relations are so important that we write the separate equations in the expanded form

$$V_a^{(0)} = \tfrac{1}{3}(V_a + V_b + V_c) \tag{11.12}$$

$$V_a^{(1)} = \tfrac{1}{3}(V_a + aV_b + a^2V_c) \tag{11.13}$$

$$V_a^{(2)} = \tfrac{1}{3}(V_a + a^2V_b + aV_c) \tag{11.14}$$

If required, the components $V_b^{(0)}$, $V_b^{(1)}$, $V_b^{(2)}$, $V_c^{(0)}$, $V_c^{(1)}$, and $V_c^{(2)}$ can be found by Eqs. (11.4). Similar results apply to line-to-line voltages simply by replacing V_a, V_b, and V_c in above equations by V_{ab}, V_{bc}, and V_{ca}, respectively.

Equation (11.12) shows that no zero-sequence components exist if the sum of the unbalanced phasors is zero. Since the sum of the line-to-line voltage phasors in a three-phase system is always zero, zero-sequence components are never present in the line voltages regardless of the degree of unbalance. The sum of the three line-to-line neutral voltage phasors is not necessarily zero, and voltages to neutral may contain zero-sequence components.

The preceding equations could have been written for any set of related phasors, and we might have written them for currents instead of for voltages. They may be solved either analytically or graphically. Because some of the preceding equations are so fundamental, they are summarized for currents:

$$
\begin{aligned}
I_a &= I_a^{(0)} + I_a^{(1)} + I_a^{(2)} \\
I_b &= I_a^{(0)} + a^2 I_a^{(1)} + a I_a^{(2)} \\
I_c &= I_a^{(0)} + a I_a^{(1)} + a^2 I_a^{(2)}
\end{aligned}
\tag{11.15}
$$

$$
\begin{aligned}
I_a^{(0)} &= \tfrac{1}{3}(I_a + I_b + I_c) \\
I_a^{(1)} &= \tfrac{1}{3}(I_a + aI_b + a^2 I_c) \\
I_a^{(2)} &= \tfrac{1}{3}(I_a + a^2 I_b + aI_c)
\end{aligned}
\tag{11.16}
$$

Finally, these results can be extended to phase currents of a Δ circuit [such as that of Fig. 11.4(a)] by replacing I_a, I_b, and I_c by I_{ab}, I_{bc}, and I_{ca}, respectively.

Example 11.1. One conductor of a three-phase line is open. The current flowing to the Δ-connected load through line a is 10 A. With the current in line a as reference and assuming that line c is open, find the symmetrical components of the line currents.

Solution. Figure 11.3 is a diagram of the circuit. The line currents are

$$I_a = 10\angle 0^\circ \text{ A} \qquad I_b = 10\angle 180^\circ \text{ A} \qquad I_c = 0 \text{ A}$$

From Eqs. (11.16)

$$I_a^{(0)} = \tfrac{1}{3}\left(10\angle 0^\circ + 10\angle 180^\circ + 0\right) = 0$$

$$
\begin{aligned}
I_a^{(1)} &= \tfrac{1}{3}\left(10\angle 0^\circ + 10\angle 180^\circ + 120^\circ + 0\right) \\
&= 5 - j2.89 = 5.78\angle -30^\circ \text{ A}
\end{aligned}
$$

$$
\begin{aligned}
I_a^{(2)} &= \tfrac{1}{3}\left(10\angle 0 + 10\angle 180^\circ + 240^\circ + 0\right) \\
&= 5 + j2.89 = 5.78\angle 30^\circ \text{ A}
\end{aligned}
$$

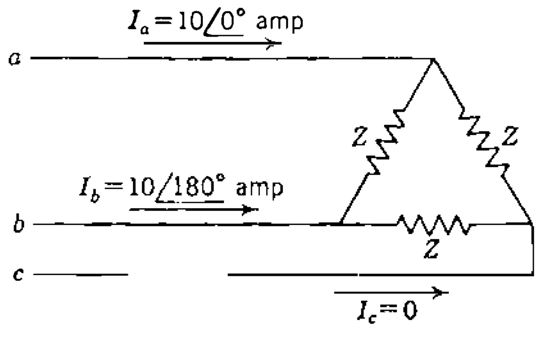


FIGURE 11.3
Circuit for Example 11.1.

From Eqs. (11.4)

$$I_b^{(0)} = 0 \qquad\qquad I_c^{(0)} = 0$$

$$I_b^{(1)} = 5.78\angle -150^\circ \text{ A} \qquad I_c^{(1)} = 5.78\angle 90^\circ \text{ A}$$

$$I_b^{(2)} = 5.78\angle 150^\circ \text{ A} \qquad I_c^{(2)} = 5.78\angle -90^\circ \text{ A}$$

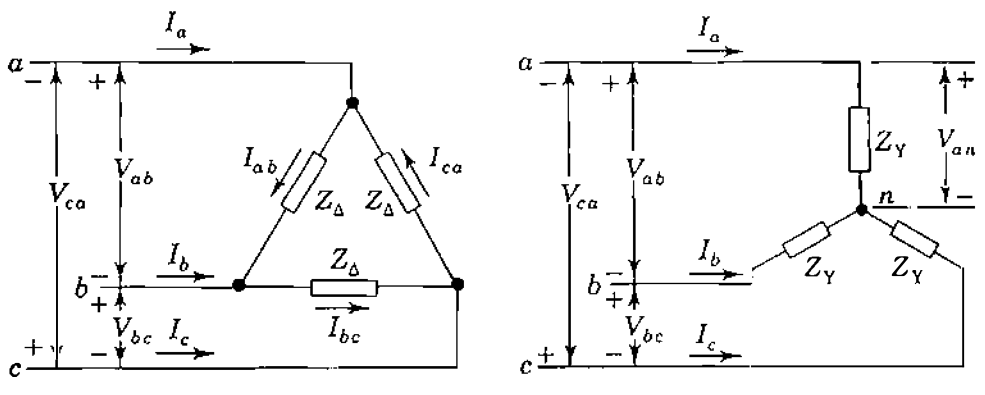
The result $I_a^{(0)} = I_b^{(0)} = I_c^{(0)} = 0$ holds for any three-wire system.

In Example 11.1 we note that components $I_c^{(1)}$ and $I_c^{(2)}$ have nonzero values although line c is open and can carry no net current. As is expected, therefore, the sum of the components in line c is zero. Of course, the sum of the components in line a is $10\angle 0^\circ$ A, and the sum of the components in line b is $10\angle 180^\circ$ A.

11.3 SYMMETRICAL Y AND Δ CIRCUITS

In three-phase systems circuit elements are connected between lines a, b, and c in either Y or Δ configuration. Relationships between the symmetrical components of Y and Δ currents and voltages can be established by referring to Fig. 11.4, which shows *symmetrical* impedances connected in Y and Δ. Let us agree that the reference phase for Δ quantities is branch a–b. The particular choice of reference phase is arbitrary and does not affect the results. For currents we have

$$\begin{aligned} I_a &= I_{ab} - I_{ca} \\ I_b &= I_{bc} - I_{ab} \\ I_c &= I_{ca} - I_{bc} \end{aligned} \tag{11.17}$$



(a) (b)

FIGURE 11.4
Symmetrical impedances: (a) Δ-connected; (b) Y-connected.

Adding all three equations together and invoking the definition of zero-sequence current, we obtain $I_a^{(0)} = (I_a + I_b + I_c)/3 = 0$, which means that *line currents into a Δ-connected circuit have no zero-sequence currents.* Substituting components of current in the equation for I_a yields

$$I_a^{(1)} + I_a^{(2)} = \left(I_{ab}^{(0)} + I_{ab}^{(1)} + I_{ab}^{(2)}\right) - \left(I_{ca}^{(0)} + I_{ca}^{(1)} + I_{ca}^{(2)}\right)$$

$$= \underbrace{\left(I_{ab}^{(0)} - I_{ca}^{(0)}\right)}_{0} + \left(I_{ab}^{(1)} - I_{ca}^{(1)}\right) + \left(I_{ab}^{(2)} - I_{ca}^{(2)}\right) \tag{11.18}$$

Evidently, if a nonzero value of circulating current $I_{ab}^{(0)}$ exists in the Δ circuit, it cannot be determined from the line currents alone. Noting that $I_{ca}^{(1)} = aI_{ab}^{(1)}$ and $I_{ca}^{(2)} = a^2I_{ab}^{(2)}$, we now write Eq. (11.18) as follows:

$$I_a^{(1)} + I_a^{(2)} = (1 - a)I_{ab}^{(1)} + (1 - a^2)I_{ab}^{(2)} \tag{11.19}$$

A similar equation for phase b is $I_b^{(1)} + I_b^{(2)} = (1 - a)I_{bc}^{(1)} + (1 - a^2)I_{bc}^{(2)}$, and expressing $I_b^{(1)}$, $I_b^{(2)}$, $I_{bc}^{(1)}$, and $I_{bc}^{(2)}$ in terms of $I_a^{(1)}$, $I_a^{(2)}$, $I_{ab}^{(1)}$, and $I_{ab}^{(2)}$, we obtain a resultant equation which can be solved along with Eq. (11.19) to yield the important results

$$I_a^{(1)} = \sqrt{3}\,\underline{/-30^\circ} \times I_{ab}^{(1)} \qquad I_a^{(2)} = \sqrt{3}\,\underline{/30^\circ} \times I_{ab}^{(2)} \tag{11.20}$$

These results amount to equating currents of the same sequence in Eq. (11.19). Complete sets of positive- and negative-sequence components of currents are shown in the phasor diagram of Fig. 11.5(a).

In a similar manner, the line-to-line voltages can be written in terms of line-to-neutral voltages of a Y-connected system,

$$\begin{aligned} V_{ab} &= V_{an} - V_{bn} \\ V_{bc} &= V_{bn} - V_{cn} \\ V_{ca} &= V_{cn} - V_{an} \end{aligned} \tag{11.21}$$

Adding together all three equations shows that $V_{ab}^{(0)} = (V_{ab} + V_{bc} + V_{ca})/3 = 0$. In words, *line-to-line voltages have no zero-sequence components.* Substituting components of the voltages in the equation for V_{ab} yields

$$V_{ab}^{(1)} + V_{ab}^{(2)} = \left(V_{an}^{(0)} + V_{an}^{(1)} + V_{an}^{(2)}\right) - \left(V_{bn}^{(0)} + V_{bn}^{(1)} + V_{bn}^{(2)}\right)$$

$$= \underbrace{\left(V_{an}^{(0)} - V_{bn}^{(0)}\right)}_{0} + \left(V_{an}^{(1)} - V_{bn}^{(1)}\right) + \left(V_{an}^{(2)} - V_{bn}^{(2)}\right) \tag{11.22}$$

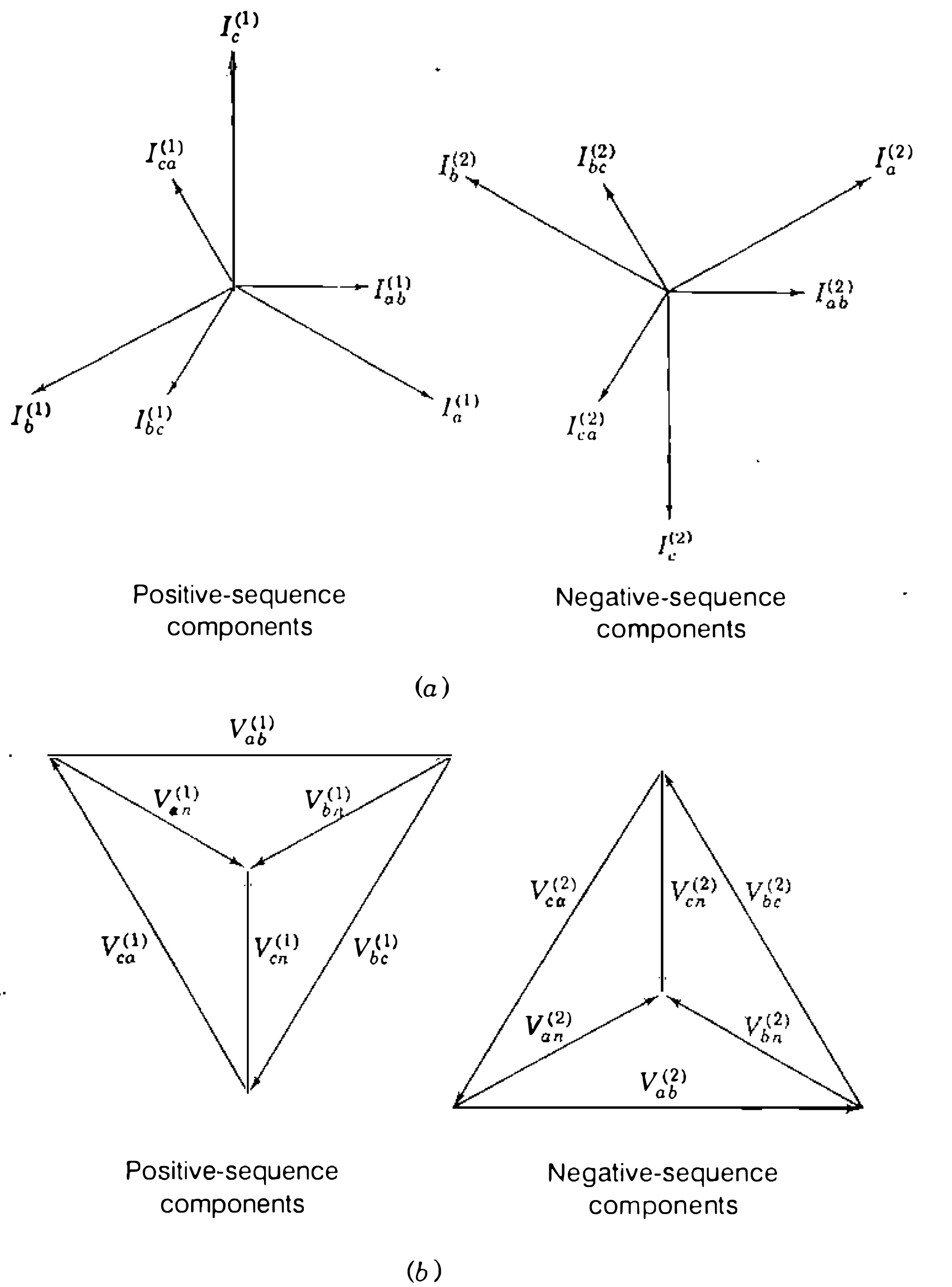


FIGURE 11.5
Positive- and negative-sequence components of (*a*) line and delta currents and (*b*) line-to-line and line-to-neutral voltages of a three-phase system.

Therefore, a nonzero value of the zero-sequence voltage $V_{an}^{(0)}$ cannot be determined from the line-to-line voltages alone. Separating positive- and negative-sequence quantities in the manner explained for Eq. (11.19), we obtain the important voltage relations

$$\begin{aligned} V_{ab}^{(1)} &= (1 - a^2)V_{an}^{(1)} = \sqrt{3}\angle 30^\circ \times V_{an}^{(1)} \\ V_{ab}^{(2)} &= (1 - a)V_{an}^{(2)} = \sqrt{3}\angle -30^\circ \times V_{an}^{(2)} \end{aligned} \tag{11.23}$$

Complete sets of positive- and negative-sequence components of voltages are shown in the phasor diagrams of Fig. 11.5(*b*). If the voltages to neutral are in per unit referred to the base voltage to neutral and the line voltages are in per unit referred to the base voltage from line to line, the $\sqrt{3}$ multipliers must be omitted from Eqs. (11.23). If both voltages are referred to the *same* base, however, the equations are correct as given. Similarly, when line and Δ currents are expressed in per unit, each on its own base, the $\sqrt{3}$ in Eqs. (11.20) disappears since the two bases are related to one another in the ratio of $\sqrt{3}:1$. When the currents are expressed on the same base, the equation is correct as written.

From Fig. 11.4 we note that $V_{ab}/I_{ab} = Z_\Delta$ when there are no sources or mutual coupling inside the Δ circuit. When positive- and negative-sequence quantities are both present, we have

$$\frac{V_{ab}^{(1)}}{I_{ab}^{(1)}} = Z_\Delta = \frac{V_{ab}^{(2)}}{I_{ab}^{(2)}} \tag{11.24}$$

Substituting from Eqs. (11.20) and (11.23), we obtain

$$\frac{\sqrt{3}\,V_{an}^{(1)}\angle 30^\circ}{\dfrac{I_a^{(1)}}{\sqrt{3}}\angle 30^\circ} = Z_\Delta = \frac{\sqrt{3}\,V_{an}^{(2)}\angle -30^\circ}{\dfrac{I_a^{(2)}}{\sqrt{3}}\angle -30^\circ}$$

so that

$$\frac{V_{an}^{(1)}}{I_a^{(1)}} = \frac{Z_\Delta}{3} = \frac{V_{an}^{(2)}}{I_a^{(2)}} \tag{11.25}$$

which shows that the Δ-connected impedances Z_Δ are equivalent to the *per-phase* or Y-connected impedances $Z_Y = Z_\Delta/3$ of Fig. 11.6(*a*) insofar as posi-

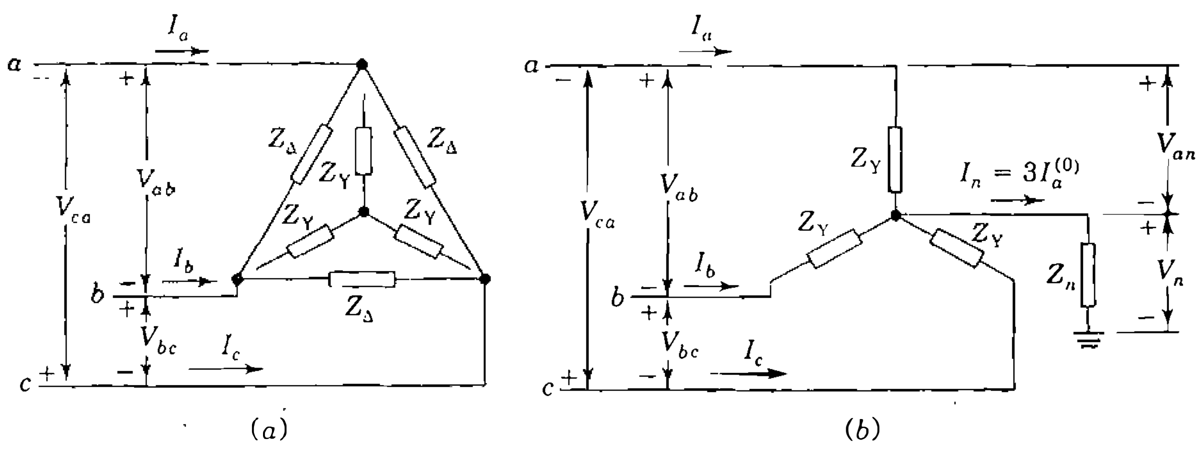


FIGURE 11.6
(*a*) Symmetrical Δ-connected impedances and their Y-connected equivalents related by $Z_Y = Z_\Delta/3$; (*b*) Y-connected impedances with neutral connection to ground.

tive- or negative-sequence currents are concerned. Of course, this result could have been anticipated from the usual Δ-Y transformations of Table 1.2. The relation $Z_Y = Z_\Delta/3$ is correct when the impedances Z_Δ and Z_Y are both expressed in ohms or in per unit on the same kilovoltampere and voltage bases.

Example 11.2. Three identical Y-connected resistors form a load bank with a three-phase rating of 2300 V and 500 kVA. If the load bank has applied voltages

$$|V_{ab}| = 1840 \text{ V} \qquad |V_{bc}| = 2760 \text{ V} \qquad |V_{ca}| = 2300 \text{ V}$$

find the line voltages and currents in per unit into the load. Assume that the neutral of the load is not connected to the neutral of the system and select a base of 2300 V, 500 kVA.

Solution. The rating of the load bank coincides with the specified base, and so the resistance values are 1.0 per unit. On the same base the given line voltages in per unit are

$$|V_{ab}| = 0.8 \qquad |V_{bc}| = 1.2 \qquad |V_{ca}| = 1.0$$

Assuming an angle of 180° for V_{ca} and using the law of cosines to find the angles of the other line voltages, we find the per-unit values

$$V_{ab} = 0.8\angle 82.8^\circ \qquad V_{bc} = 1.2\angle -41.4^\circ \qquad V_{ca} = 1.0\angle 180^\circ$$

The symmetrical components of the line voltages are

$$V_{ab}^{(1)} = \tfrac{1}{3}\left(0.8\angle 82.8^\circ + 1.2\angle 120^\circ - 41.4^\circ + 1.0\angle 240^\circ + 180^\circ\right)$$

$$= \tfrac{1}{3}(0.1003 + j0.7937 + 0.2372 + j1.1763 + 0.5 + j0.8660)$$

$$= 0.2792 + j0.9453 = 0.9857\angle 73.6^\circ \text{ per unit (line-to-line voltage base)}$$

$$V_{ab}^{(2)} = \tfrac{1}{3}\left(0.8\angle 82.8^\circ + 1.2\angle 240^\circ - 41.4^\circ + 1.0\angle 120^\circ + 180^\circ\right)$$

$$= \tfrac{1}{3}(0.1003 + j0.7937 - 1.1373 - j0.3828 + 0.5 - j0.8660)$$

$$= -0.1790 - j0.1517 = 0.2346\angle 220.3^\circ \text{ per unit (line-to-line voltage base)}$$

The absence of a neutral connection means that zero-sequence currents are not present. Therefore, the phase voltages at the load contain positive- and negative-sequence components only. The phase voltages are found from Eqs. (11.23) with the $\sqrt{3}$ factor omitted since the line voltages are expressed in terms of the base voltage from line to line and the phase voltages are desired in per unit of the base voltage to neutral. Thus,

$$V_{an}^{(1)} = 0.9857\angle 73.6^\circ - 30^\circ$$

$$= 0.9857\angle 43.6^\circ \text{ per unit (line-to-neutral voltage base)}$$

$$V_{an}^{(2)} = 0.2346\angle 220.3^\circ + 30^\circ$$

$$= 0.2346\angle 250.3^\circ \text{ per unit (line-to-neutral voltage base)}$$

Since each resistor has an impedance of $1.0\angle 0^\circ$ per unit,

$$I_a^{(1)} = \frac{V_a^{(1)}}{1.0\angle 0^\circ} = 0.9857\angle 43.6^\circ \text{ per unit}$$

$$I_a^{(2)} = \frac{V_a^{(2)}}{1.0\angle 0^\circ} = 0.2346\angle 250.3^\circ \text{ per unit}$$

The positive direction of current is chosen to be from the supply toward the load.

11.4 POWER IN TERMS OF SYMMETRICAL COMPONENTS

If the symmetrical components of current and voltage are known, the power expended in a three-phase circuit can be computed directly from the components. Demonstration of this statement is a good example of the matrix manipulation of symmetrical components.

The total complex power flowing into a three-phase circuit through three lines a, b, and c is

$$S_{3\phi} = P + jQ = V_a I_a^* + V_b I_b^* + V_c I_c^* \tag{11.26}$$

where V_a, V_b, and V_c are the voltages to reference at the terminals and I_a, I_b, and I_c are the currents flowing into the circuit in the three lines. A neutral connection may or may not be present. If there is impedance in the neutral connection to ground, then the voltages V_a, V_b, and V_c must be interpreted as voltages from the line to ground rather than to neutral. In matrix notation

$$S_{3\phi} = \begin{bmatrix} V_a & V_b & V_c \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}^* = \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix}^T \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}^* \tag{11.27}$$

where the conjugate of a matrix is understood to be composed of elements that are the conjugates of the corresponding elements of the original matrix.

To introduce the symmetrical components of the voltages and currents, we make use of Eq. (11.8) to obtain

$$S_{3\phi} = [\mathbf{A}\mathbf{V}_{012}]^T[\mathbf{A}\mathbf{I}_{012}]^* \tag{11.28}$$

where

$$\mathbf{V}_{012} = \begin{bmatrix} V_a^{(0)} \\ V_a^{(1)} \\ V_a^{(2)} \end{bmatrix} \quad \text{and} \quad \mathbf{I}_{012} = \begin{bmatrix} I_a^{(0)} \\ I_a^{(1)} \\ I_a^{(2)} \end{bmatrix} \tag{11.29}$$

The *reversal rule* of matrix algebra states that the transpose of the product of two matrices is equal to the product of the transposes of the matrices in reverse order. According to this rule,

$$[\mathbf{A}\mathbf{V}_{012}]^T = \mathbf{V}_{012}^T\mathbf{A}^T \tag{11.30}$$

and so

$$S_{3\phi} = \mathbf{V}_{012}^T\mathbf{A}^T[\mathbf{A}\mathbf{I}_{012}]^* = \mathbf{V}_{012}^T\mathbf{A}^T\mathbf{A}^*\mathbf{I}_{012}^* \tag{11.31}$$

Noting that $\mathbf{A}^T = \mathbf{A}$ and that a and a^2 are conjugates, we obtain

$$S_{3\phi} = \begin{bmatrix} V_a^{(0)} & V_a^{(1)} & V_a^{(2)} \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} I_a^{(0)} \\ I_a^{(1)} \\ I_a^{(2)} \end{bmatrix}^* \tag{11.32}$$

or since

$$\mathbf{A}^T\mathbf{A}^* = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} = 3\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$S_{3\phi} = 3\begin{bmatrix} V_a^{(0)} & V_a^{(1)} & V_a^{(2)} \end{bmatrix} \begin{bmatrix} I_a^{(0)} \\ I_a^{(1)} \\ I_a^{(2)} \end{bmatrix}^* \tag{11.33}$$

So, complex power is

$$S_{3\phi} = V_a I_a^* + V_b I_b^* + V_c I_c^* = 3V_a^{(0)}I_a^{(0)*} + 3V_a^{(1)}I_a^{(1)*} + 3V_a^{(2)}I_a^{(2)*} \tag{11.34}$$

which shows how complex power (in *voltamperes*) can be computed from the symmetrical components of the voltages to reference (in *volts*) and line currents (in *amperes*) of an unbalanced three-phase circuit. It is important to note that the transformation of *a-b-c* voltages and currents to symmetrical components is power-invariant in the sense discussed in Sec. 8.6, only if each product of sequence voltage (in volts) times the complex conjugate of the corresponding sequence current (in amperes) is multiplied by 3, as shown in Eq. (11.34). When the complex power $S_{3\phi}$ is expressed in per unit of a three-phase voltampere base, however, the multiplier 3 disappears.

Example 11.3. Using symmetrical components, calculate the power absorbed in the load of Example 11.2 and check the answer.

Solution. In per unit of the three-phase 500-kVA base, Eq. (11.34) becomes

$$S_{3\phi} = V_a^{(0)} I_a^{(0)*} + V_a^{(1)} I_a^{(1)*} + V_a^{(2)} I_a^{(2)*}$$

Substituting the components of voltages and currents from Example 11.2, we obtain

$$S_{3\phi} = 0 + 0.9857\underline{/43.6^\circ} \times 0.9857\underline{/-43.6^\circ} + 0.2346\underline{/250.3^\circ} \times 0.2346\underline{/-250.3^\circ}$$

$$= (0.9857)^2 + (0.2346)^2 = 1.02664 \text{ per unit}$$

$$= 513.32 \text{ kW}$$

The per-unit value of the resistors in each phase of the Y-connected load bank is 1.0 per unit. In ohms, therefore,

$$R_Y = \frac{(2300)^2}{500{,}000} = 10.58\ \Omega$$

and the equivalent Δ-connected resistors are

$$R_\Delta = 3R_Y = 31.74\ \Omega$$

From the given line-to-line voltages we calculate directly

$$S_{3\phi} = \frac{|V_{ab}|^2}{R_\Delta} + \frac{|V_{bc}|^2}{R_\Delta} + \frac{|V_{ca}|^2}{R_\Delta}$$

$$= \frac{(1840)^2 + (2760)^2 + (2300)^2}{31.74} = 513.33 \text{ kW}$$

11.5 SEQUENCE CIRCUITS OF Y AND Δ IMPEDANCES

If impedance Z_n is inserted between the neutral and ground of the Y-connected impedances shown in Fig. 11.6(*b*), then the sum of the line currents is equal to the current I_n in the return path through the neutral. That is,

$$I_n = I_a + I_b + I_c \tag{11.35}$$

Expressing the unbalanced line currents in terms of their symmetrical components gives

$$\begin{aligned} I_n &= \left(I_a^{(0)} + I_a^{(1)} + I_a^{(2)}\right) + \left(I_b^{(0)} + I_b^{(1)} + I_b^{(2)}\right) + \left(I_c^{(0)} + I_c^{(1)} + I_c^{(2)}\right) \\ &= \left(I_a^{(0)} + I_b^{(0)} + I_c^{(0)}\right) + \underbrace{\left(I_a^{(1)} + I_b^{(1)} + I_c^{(1)}\right)}_{0} + \underbrace{\left(I_a^{(2)} + I_b^{(2)} + I_c^{(2)}\right)}_{0} \\ &= 3I_a^{(0)} \end{aligned} \tag{11.36}$$

Since the positive-sequence and negative-sequence currents add separately to zero at neutral point n, there cannot be any positive-sequence or negative-sequence currents in the connections from neutral to ground regardless of the value of Z_n. Moreover, the zero-sequence currents combining together at n become $3I_a^{(0)}$, which produces the voltage drop $3I_a^{(0)}Z_n$ between neutral and ground. It is important, therefore, to distinguish between voltages to neutral and voltages to ground under unbalanced conditions. Let us designate voltages of phase a with respect to neutral and ground as V_{an} and V_a, respectively. Thus, the voltage of phase a with respect to ground is given by $V_a = V_{an} + V_n$, where $V_n = 3I_a^{(0)}Z_n$. Referring to Fig. 11.6(b), we can write the voltage drops to ground from each of the lines a, b, and c as

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} V_{an} \\ V_{bn} \\ V_{cn} \end{bmatrix} + \begin{bmatrix} V_n \\ V_n \\ V_n \end{bmatrix} = Z_Y \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} + 3I_a^{(0)}Z_n \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \tag{11.37}$$

The a-b-c voltages and currents in this equation can be replaced by their symmetrical components as follows:

$$\mathbf{A} \begin{bmatrix} V_a^{(0)} \\ V_a^{(1)} \\ V_a^{(2)} \end{bmatrix} = Z_Y \mathbf{A} \begin{bmatrix} I_a^{(0)} \\ I_a^{(1)} \\ I_a^{(2)} \end{bmatrix} + 3I_a^{(0)}Z_n \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \tag{11.38}$$

Multiplying across by the inverse matrix $\mathbf{A}^{-1}$, we obtain

$$\begin{bmatrix} V_a^{(0)} \\ V_a^{(1)} \\ V_a^{(2)} \end{bmatrix} = Z_Y \begin{bmatrix} I_a^{(0)} \\ I_a^{(1)} \\ I_a^{(2)} \end{bmatrix} + 3I_a^{(0)}Z_n \mathbf{A}^{-1} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

Postmultiplying $\mathbf{A}^{-1}$ by $[1 \quad 1 \quad 1]^T$ amounts to adding the elements in each row of $\mathbf{A}^{-1}$,

and so
$$\begin{bmatrix} V_a^{(0)} \\ V_a^{(1)} \\ V_a^{(2)} \end{bmatrix} = Z_Y \begin{bmatrix} I_a^{(0)} \\ I_a^{(1)} \\ I_a^{(2)} \end{bmatrix} + 3I_a^{(0)}Z_n \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \tag{11.39}$$

In expanded form, Eq. (11.39) becomes three separate or *decoupled* equations,

$$V_a^{(0)} = (Z_Y + 3Z_n) I_a^{(0)} = Z_0 I_a^{(0)} \tag{11.40}$$

$$V_a^{(1)} = Z_Y I_a^{(1)} = Z_1 I_a^{(1)} \tag{11.41}$$

$$V_a^{(2)} = Z_Y I_a^{(2)} = Z_2 I_a^{(2)} \tag{11.42}$$

It is customary to use the symbols Z_0, Z_1, and Z_2 as shown.

Equations (11.40) through (11.42) could have been easily developed in a less formal manner, but the matrix approach adopted here will be useful in developing other important relations in the sections which follow. Equations (11.24) and (11.25) combine with Eqs. (11.40) through (11.42) to show that currents of one sequence cause voltage drops of only the *same* sequence in Δ- or Y-connected circuits with symmetrical impedances in each phase. This most important result allows us to draw the three single-phase *sequence circuits* shown in Fig. 11.7. These three circuits, considered simultaneously, provide the same information as the actual circuit of Fig. 11.6(*b*), and they are independent of one another because Eqs. (11.40) through (11.42) are decoupled. The circuit of Fig. 11.7(*a*) is called the *zero-sequence circuit* because it relates the zero-sequence voltage $V_a^{(0)}$ to the zero-sequence current $I_a^{(0)}$, and thereby serves to define the *impedance to zero-sequence current* given by

$$\frac{V_a^{(0)}}{I_a^{(0)}} = Z_0 = Z_Y + 3Z_n \tag{11.43}$$

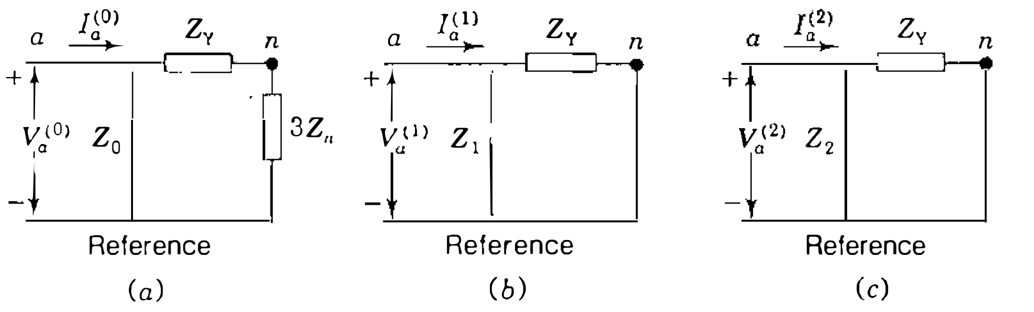


FIGURE 11.7
Zero-, positive-, and negative-sequence circuits for Fig. 11.6(*b*).

Likewise, Fig. 11.7(*b*) is called the *positive-sequence circuit* and Z_1 is called the *impedance to positive-sequence current*, whereas Fig. 11.7(*c*) is the *negative-sequence circuit* and Z_2 is the *impedance to negative-sequence current*. The names of the impedances to currents of the different sequences are usually shortened to the less descriptive terms *zero-sequence impedance* Z_0, *positive-sequence impedance* Z_1, and *negative-sequence impedance* Z_2. Here the positive- and negative-sequence impedances Z_1 and Z_2, respectively, are both found to be equal to the usual per-phase impedance Z_Y, which is generally the case for stationary symmetrical circuits. Each of the three sequence circuits represents one phase of the actual three-phase circuit when the latter carries current of only that sequence. When the three sequence currents are simultaneously present, *all three* sequence circuits are needed to fully represent the original circuit.

Voltages in the positive-sequence and negative-sequence circuits can be regarded as voltages measured with respect to either neutral or ground whether or not there is a connection of some finite value of impedance Z_n between neutral and ground. Accordingly, in the positive-sequence circuit there is no difference between $V_a^{(1)}$ and $V_{an}^{(1)}$, and a similar statement applies to $V_a^{(2)}$ and $V_{an}^{(2)}$ in the negative-sequence circuit. However, a voltage difference can exist between the neutral and the reference of the zero-sequence circuit. In the circuit of Fig. 11.7(*a*) the current $I_a^{(0)}$ flowing through impedance $3Z_n$ produces the same voltage drop from neutral to ground as the current $3I_a^{(0)}$ flowing through impedance Z_n in the actual circuit of Fig. 11.6(*b*).

If the neutral of the Y-connected circuit is grounded through zero impedance, we set $Z_n = 0$ and a zero-impedance connection then joins the neutral point to the reference node of the zero-sequence circuit. If there is no connection between neutral and ground, there cannot be any zero-sequence current flow, for then $Z_n = \infty$, which is indicated by the open circuit between neutral and the reference node in the zero-sequence circuit of Fig. 11.8(*a*).

Obviously, a Δ-connected circuit cannot provide a path through neutral, and so line currents flowing into a Δ-connected load or its equivalent Y circuit cannot contain any zero-sequence components. Consider the symmetrical Δ-connected circuit of Fig. 11.4 with

$$V_{ab} = Z_\Delta I_{ab} \qquad V_{bc} = Z_\Delta I_{bc} \qquad V_{ca} = Z_\Delta I_{ca} \tag{11.44}$$

Adding the three preceding equations together, we obtain

$$V_{ab} + V_{bc} + V_{ca} = 3V_{ab}^{(0)} = 3Z_\Delta I_{ab}^{(0)} \tag{11.45}$$

and since the sum of the line-to-line voltages is always zero, we therefore have

$$V_{ab}^{(0)} = I_{ab}^{(0)} = 0 \tag{11.46}$$

Thus, in Δ-connected circuits with impedances only and no sources or mutual coupling there cannot be any circulating currents. Sometimes single-phase

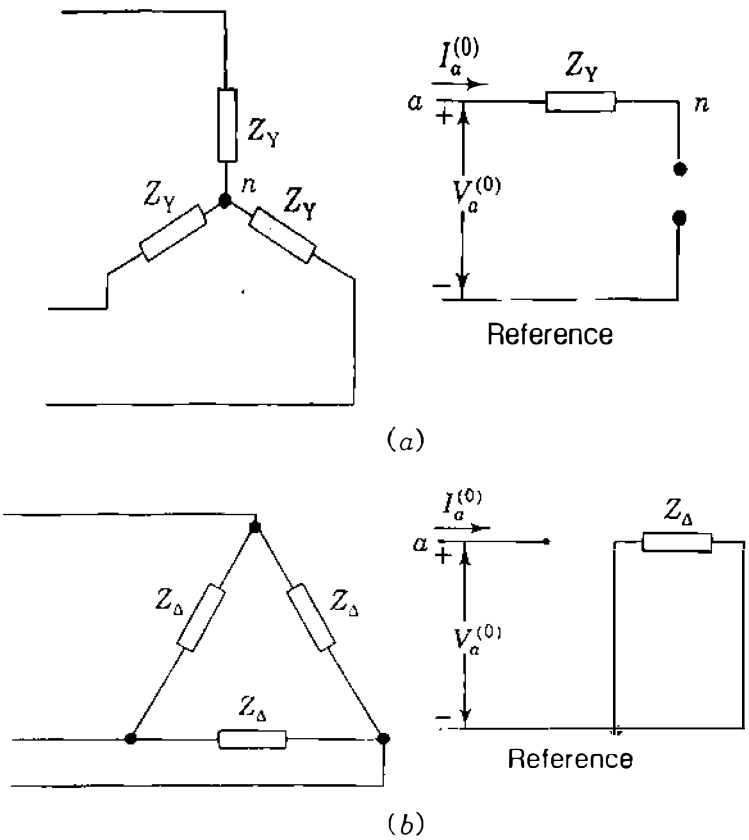


FIGURE 11.8
(*a*) Ungrounded Y-connected and (*b*) Δ-connected circuits and their zero-sequence circuits.

circulating currents can be produced in the Δ circuits of transformers and generators by either induction or zero-sequence generated voltages. A Δ circuit and its zero-sequence circuit are shown in Fig. 11.8(*b*). Note, however, that even if zero-sequence voltages were generated in the phases of the Δ, no zero-sequence voltage could exist between the Δ terminals, for the *rise* in voltage in each phase would then be matched by the voltage *drop* in the zero-sequence impedance of each phase.

Example 11.4. Three equal impedances of $j21\ \Omega$ are connected in Δ. Determine the sequence impedances and circuits of the combination. Repeat the solution for the case where a mutual impedance of $j6\ \Omega$ exists between each pair of adjacent branches in the Δ.

Solution. The line-to-line voltages are related to the Δ currents by

$$\begin{bmatrix} V_{ab} \\ V_{bc} \\ V_{ca} \end{bmatrix} = \begin{bmatrix} j21 & 0 & 0 \\ 0 & j21 & 0 \\ 0 & 0 & j21 \end{bmatrix} \begin{bmatrix} I_{ab} \\ I_{bc} \\ I_{ca} \end{bmatrix}$$

Transforming to symmetrical components of voltages and currents gives

$$\mathbf{A}\begin{bmatrix} V_{ab}^{(0)} \\ V_{ab}^{(1)} \\ V_{ab}^{(2)} \end{bmatrix} = \begin{bmatrix} j21 & 0 & 0 \\ 0 & j21 & 0 \\ 0 & 0 & j21 \end{bmatrix} \mathbf{A} \begin{bmatrix} I_{ab}^{(0)} \\ I_{ab}^{(1)} \\ I_{ab}^{(2)} \end{bmatrix}$$

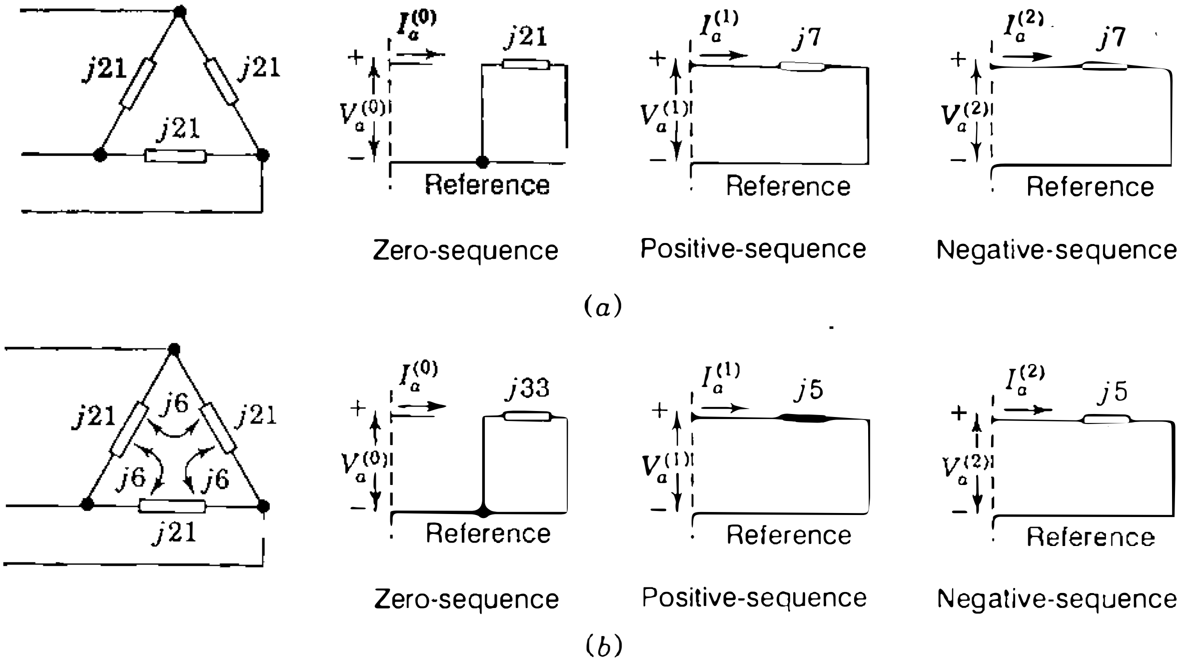


FIGURE 11.9
Zero-, positive-, and negative-sequence circuits for Δ-connected impedances of Example 11.4.

and premultiplying each side by $\mathbf{A}^{-1}$, we obtain

$$\begin{bmatrix} V_{ab}^{(0)} \\ V_{ab}^{(1)} \\ V_{ab}^{(2)} \end{bmatrix} = j21\mathbf{A}^{-1}\mathbf{A}\begin{bmatrix} I_{ab}^{(0)} \\ I_{ab}^{(1)} \\ I_{ab}^{(2)} \end{bmatrix} = \begin{bmatrix} j21 & 0 & 0 \\ 0 & j21 & 0 \\ 0 & 0 & j21 \end{bmatrix}\begin{bmatrix} I_{ab}^{(0)} \\ I_{ab}^{(1)} \\ I_{ab}^{(2)} \end{bmatrix}$$

The positive- and negative-sequence circuits have per-phase impedances $Z_1 = Z_2 = j7\ \Omega$, as shown in Fig. 11.9(*a*), and since $V_{ab}^{(0)} = 0$, the zero-sequence current $I_{ab}^{(0)} = 0$ so that the zero-sequence circuit is an open circuit. The $j21$-Ω resistance in the zero-sequence network has significance only when there is an internal source in the original Δ circuit.

When there is mutual inductance $j6\ \Omega$ between phases,

$$\mathbf{A}\begin{bmatrix} V_{ab}^{(0)} \\ V_{ab}^{(1)} \\ V_{ab}^{(2)} \end{bmatrix} = \begin{bmatrix} j21 & j6 & j6 \\ j6 & j21 & j6 \\ j6 & j6 & j21 \end{bmatrix}\mathbf{A}\begin{bmatrix} I_{ab}^{(0)} \\ I_{ab}^{(1)} \\ I_{ab}^{(2)} \end{bmatrix}$$

The coefficient matrix can be separated into two parts as follows:

$$\begin{bmatrix} j21 & j6 & j6 \\ j6 & j21 & j6 \\ j6 & j6 & j21 \end{bmatrix} = j15\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + j6\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

and substituting into the previous equation, we obtain

$$\begin{bmatrix} V_{ab}^{(0)} \\ V_{ab}^{(1)} \\ V_{ab}^{(2)} \end{bmatrix} = \left\{ j15\mathbf{A}^{-1}\mathbf{A} + j6\mathbf{A}^{-1} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \mathbf{A} \right\} \begin{bmatrix} I_{ab}^{(0)} \\ I_{ab}^{(1)} \\ I_{ab}^{(2)} \end{bmatrix}$$

$$= \left\{ \begin{bmatrix} j15 & 0 & 0 \\ 0 & j15 & 0 \\ 0 & 0 & j15 \end{bmatrix} + j6 \begin{bmatrix} 3 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \right\} \begin{bmatrix} I_{ab}^{(0)} \\ I_{ab}^{(1)} \\ I_{ab}^{(2)} \end{bmatrix}$$

$$= \begin{bmatrix} j33 & 0 & 0 \\ 0 & j15 & 0 \\ 0 & 0 & j15 \end{bmatrix} \begin{bmatrix} I_{ab}^{(0)} \\ I_{ab}^{(1)} \\ I_{ab}^{(2)} \end{bmatrix}$$

The positive- and negative-sequence impedances Z_1 and Z_2 now take on the value $j5\ \Omega$, as shown in Fig. 11.9(*b*), and since $V_{ab}^{(0)} = I_{ab}^{(0)} = 0$, the zero-sequence circuit is open. Again, we note that the $j33$-Ω resistance in the zero-sequence network has no significance because there is no internal source in the original Δ circuit.

The matrix manipulations of this example are useful in the sections which follow.

11.6 SEQUENCE CIRCUITS OF A SYMMETRICAL TRANSMISSION LINE

We are concerned primarily with systems that are essentially symmetrically balanced and which become unbalanced only upon the occurrence of an unsymmetrical fault. In practical transmission systems such complete symmetry is more ideal than realized, but since the effect of the departure from symmetry is usually small, perfect balance between phases is often assumed especially if the lines are transposed along their lengths. Let us consider Fig. 11.10, for instance, which shows one section of a three-phase transmission line with a neutral conductor. The self-impedance Z_{aa} is the same for each phase conductor, and the neutral conductor has self-impedance Z_{nn}. When currents I_a, I_b, and I_c in the phase conductors are unbalanced, the neutral conductor serves as a return path. All the currents are assumed positive in the directions shown even though some of their numerical values may be negative under unbalanced conditions caused by faults. Because of mutual coupling, current flow in any one of the phases induces voltages in each of the other adjacent phases and in the neutral conductor. Similarly, I_n in the neutral conductor induces voltages in each of the phases. The coupling between all three phase conductors is regarded as being symmetrical and mutual impedance Z_{ab} is assumed between

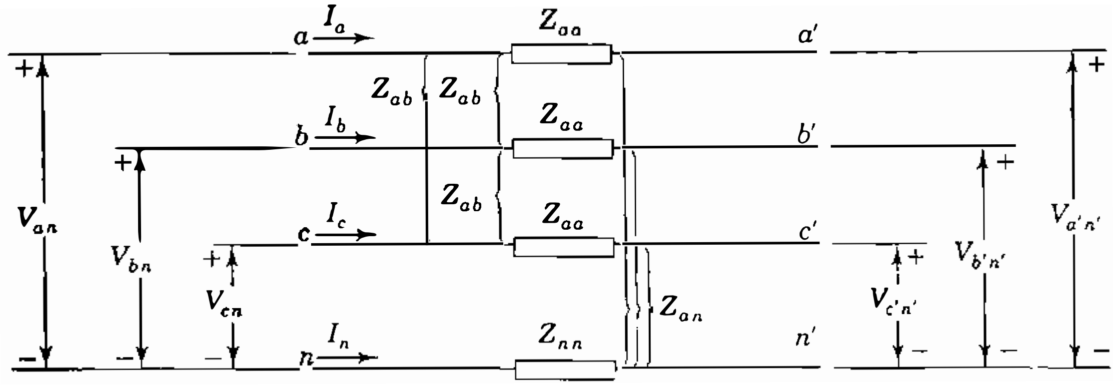


FIGURE 11.10
Flow of unbalanced currents in a symmetrical three-phase-line section with neutral conductor.

each pair. Likewise, the mutual impedance between the neutral conductor and each of the phases is taken to be Z_{an}.

The voltages induced in phase a, for example, by currents in the other two phases and the neutral conductor are shown as sources in the loop circuit of Fig. 11.11, along with the similar voltages induced in the neutral conductor. Applying Kirchhoff's voltage law around the loop circuit gives

$$\begin{aligned} V_{an} &= Z_{aa}I_a + Z_{ab}I_b + Z_{ab}I_c + Z_{an}I_n + V_{a'n'} \\ &\quad -(Z_{nn}I_n + Z_{an}I_c + Z_{an}I_b + Z_{an}I_a) \end{aligned} \tag{11.47}$$

from which voltage drop across the line section is found to be

$$V_{an} - V_{a'n'} = (Z_{aa} - Z_{an})I_a + (Z_{ab} - Z_{an})(I_b + I_c) + (Z_{an} - Z_{nn})I_n \tag{11.48}$$

Similar equations can be written for phases b and c as follows:

$$\begin{aligned} V_{bn} - V_{b'n'} &= (Z_{aa} - Z_{an})I_b + (Z_{ab} - Z_{an})(I_a + I_c) + (Z_{an} - Z_{nn})I_n \\ V_{cn} - V_{c'n'} &= (Z_{aa} - Z_{an})I_c + (Z_{ab} - Z_{an})(I_a + I_b) + (Z_{an} - Z_{nn})I_n \end{aligned} \tag{11.49}$$

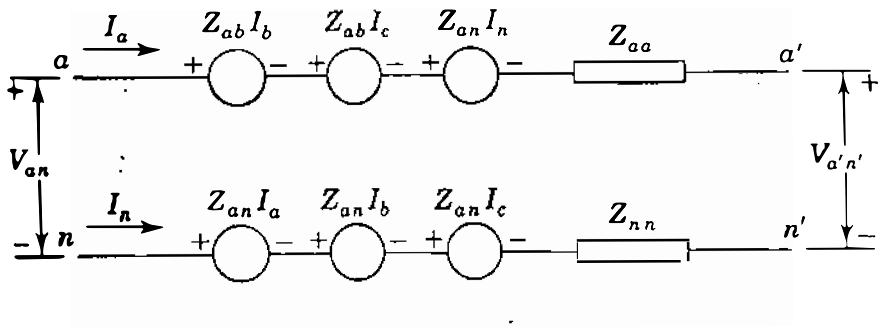


FIGURE 11.11
Writing Kirchhoff's voltage equation around the loop formed by line a and the neutral conductor.

When the line currents I_a, I_b, and I_c return together as I_n in the neutral conductor of Fig. 11.10, we have

$$I_n = -(I_a + I_b + I_c) \tag{11.50}$$

Let us now substitute for I_n in Eqs. (11.48) and (11.49) to obtain

$$\begin{aligned}
V_{an} - V_{a'n'} &= (Z_{aa} + Z_{nn} - 2Z_{an})I_a + (Z_{ab} + Z_{nn} - 2Z_{an})I_b \\
&\quad + (Z_{ab} + Z_{nn} - 2Z_{an})I_c \\
V_{bn} - V_{b'n'} &= (Z_{ab} + Z_{nn} - 2Z_{an})I_a + (Z_{aa} + Z_{nn} - 2Z_{an})I_b \\
&\quad + (Z_{ab} + Z_{nn} - 2Z_{an})I_c \\
V_{cn} - V_{c'n'} &= (Z_{ab} + Z_{nn} - 2Z_{an})I_a + (Z_{ab} + Z_{nn} - 2Z_{an})I_b \\
&\quad + (Z_{aa} + Z_{nn} - 2Z_{an})I_c
\end{aligned} \tag{11.51}$$

The coefficients in these equations show that the presence of the neutral conductor changes the self- and mutual impedances of the phase conductors to the following effective values:

$$\begin{aligned}
Z_s &\triangleq Z_{aa} + Z_{nn} - 2Z_{an} \\
Z_m &\triangleq Z_{ab} + Z_{nn} - 2Z_{an}
\end{aligned} \tag{11.52}$$

Using these definitions, we can rewrite Eqs. (11.51) in the convenient matrix form

$$\begin{bmatrix} V_{aa'} \\ V_{bb'} \\ V_{cc'} \end{bmatrix} = \begin{bmatrix} V_{an} - V_{a'n'} \\ V_{bn} - V_{b'n'} \\ V_{cn} - V_{c'n'} \end{bmatrix} = \begin{bmatrix} Z_s & Z_m & Z_m \\ Z_m & Z_s & Z_m \\ Z_m & Z_m & Z_s \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} \tag{11.53}$$

where the voltage drops across the phase conductors are now denoted by

$$V_{aa'} \triangleq V_{an} - V_{a'n'} \qquad V_{bb'} \triangleq V_{bn} - V_{b'n'} \qquad V_{cc'} \triangleq V_{cn} - V_{c'n'} \tag{11.54}$$

Since Eq. (11.53) does not *explicitly* include the neutral conductor, Z_s and Z_m can be regarded as parameters of the phase conductors alone, without any self- or mutual inductance being associated with the return path.

The *a-b-c* voltage drops and currents of the line section can be written in terms of their symmetrical components according to Eq. (11.8) so that with phase a as the reference phase, we have

$$\mathbf{A}\begin{bmatrix} V_{aa'}^{(0)} \\ V_{aa'}^{(1)} \\ V_{aa'}^{(2)} \end{bmatrix} = \left\{\begin{bmatrix} Z_s - Z_m & \cdot & \cdot \\ \cdot & Z_s - Z_m & \cdot \\ \cdot & \cdot & Z_s - Z_m \end{bmatrix} + \begin{bmatrix} Z_m & Z_m & Z_m \\ Z_m & Z_m & Z_m \\ Z_m & Z_m & Z_m \end{bmatrix}\right\}\mathbf{A}\begin{bmatrix} I_a^{(0)} \\ I_a^{(1)} \\ I_a^{(2)} \end{bmatrix} \tag{11.55}$$

This particular form of the equation makes calculations easier, as demonstrated in Example 11.4. Multiplying across by $\mathbf{A}^{-1}$, we obtain

$$\begin{bmatrix} V_{aa'}^{(0)} \\ V_{aa'}^{(1)} \\ V_{aa'}^{(2)} \end{bmatrix} = \mathbf{A}^{-1}\left\{(Z_s - Z_m)\begin{bmatrix} 1 & \cdot & \cdot \\ \cdot & 1 & \cdot \\ \cdot & \cdot & 1 \end{bmatrix} + Z_m\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}\right\}\mathbf{A}\begin{bmatrix} I_a^{(0)} \\ I_a^{(1)} \\ I_a^{(2)} \end{bmatrix} \tag{11.56}$$

The matrix multiplication here is the same as in Example 11.4 and yields

$$\begin{bmatrix} V_{aa'}^{(0)} \\ V_{aa'}^{(1)} \\ V_{aa'}^{(2)} \end{bmatrix} = \begin{bmatrix} Z_s + 2Z_m & \cdot & \cdot \\ \cdot & Z_s - Z_m & \cdot \\ \cdot & \cdot & Z_s - Z_m \end{bmatrix}\begin{bmatrix} I_a^{(0)} \\ I_a^{(1)} \\ I_a^{(2)} \end{bmatrix} \tag{11.57}$$

Let us now define zero-, positive-, and negative-sequence impedances in terms of Z_s and Z_m introduced in Eqs. (11.52),

$$\begin{aligned} Z_0 &= Z_s + 2Z_m = Z_{aa} + 2Z_{ab} + 3Z_{nn} - 6Z_{an} \\ Z_1 &= Z_s - Z_m = Z_{aa} - Z_{ab} \\ Z_2 &= Z_s - Z_m = Z_{aa} - Z_{ab} \end{aligned} \tag{11.58}$$

From Eqs. (11.57) and (11.58) the sequence components of the voltage drops between the two ends of the line section can be written as three simple equations of the form

$$\begin{aligned} V_{aa'}^{(0)} &= V_{an}^{(0)} - V_{a'n'}^{(0)} = Z_0 I_a^{(0)} \\ V_{aa'}^{(1)} &= V_{an}^{(1)} - V_{a'n'}^{(1)} = Z_1 I_a^{(1)} \\ V_{aa'}^{(2)} &= V_{an}^{(2)} - V_{a'n'}^{(2)} = Z_2 I_a^{(2)} \end{aligned} \tag{11.59}$$

Because of the assumed symmetry of the circuit of Fig. 11.10, once again we see

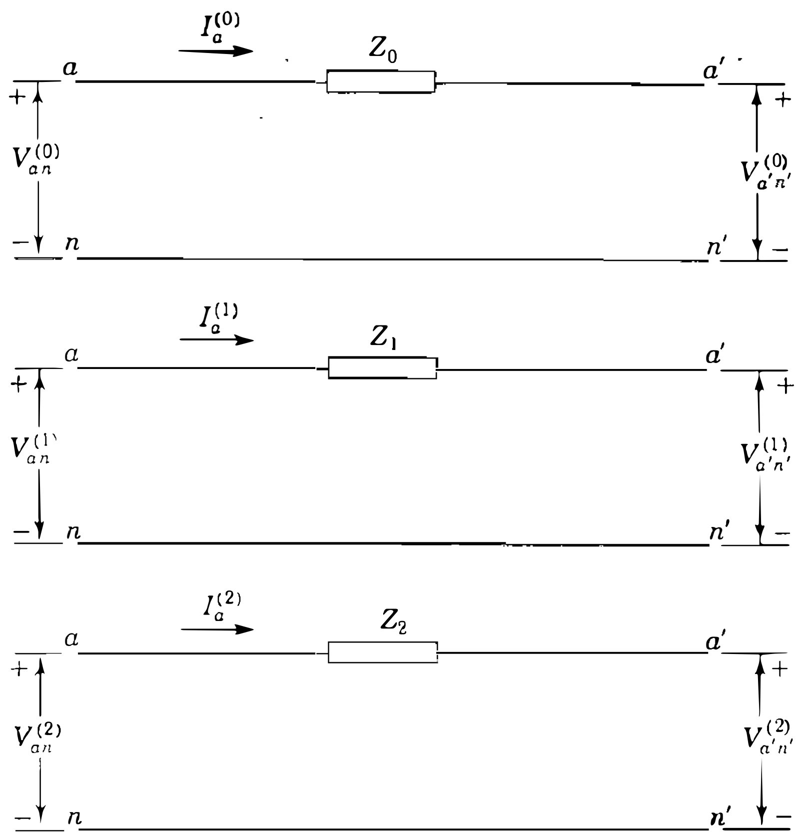


FIGURE 11.12
Sequence circuits for the symmetrical line section of Fig. 11.10.

that the zero-, positive-, and negative-sequence equations decouple from one another, and corresponding zero-, positive-, and negative-sequence circuits can be drawn without any mutual coupling between them, as shown in Fig. 11.12. Despite the simplicity of the line model in Fig. 11.10, the above development has demonstrated important characteristics of the sequence impedances which apply to more elaborate and practical line models. We note, for instance, that the positive- and negative-sequence impedances are equal and that they do not include the neutral-conductor impedances Z_{nn} and Z_{an}, which enter into the calculation of only the zero-sequence impedance Z_0, as shown by Eqs. (11.58). In other words, impedance parameters of the return-path conductors enter into the values of the zero-sequence impedances of transmission lines, but they do not affect either the positive- or negative-sequence impedance.

Most aerial transmission lines have at least two overhead conductors called *ground wires*, which are grounded at uniform intervals along the length of the line. The ground wires combine with the earth return path to constitute an effective neutral conductor with impedance parameters, like Z_{nn} and Z_{an}, which depend on the resistivity of the earth. The more specialized literature shows, as we have demonstrated here, that the parameters of the return path are included in the zero-sequence impedance of the line. By regarding the neutral conductor of Fig. 11.10 as the effective return path for the zero-sequence components of the unbalanced currents and including its parameters in the zero-sequence impedance, we can treat the ground as an ideal conductor. The voltages of Fig. 11.12 are then interpreted as being measured with respect to perfectly conducting ground, and we can write

$$V_{aa'}^{(0)} = V_a^{(0)} - V_{a'}^{(0)} = Z_0 I_a^{(0)}$$

$$V_{aa'}^{(1)} = V_a^{(1)} - V_{a'}^{(1)} = Z_1 I_a^{(1)} \qquad (11.60)$$

$$V_{aa'}^{(2)} = V_a^{(2)} - V_{a'}^{(2)} = Z_2 I_a^{(2)}$$

where the sequence components of the voltages V_a and $V_{a'}$ are now with respect to *ideal ground.*

In deriving the equations for inductance and capacitance of *transposed* transmission lines, we assumed balanced three-phase currents and did not specify phase order. The resulting parameters are therefore valid for both positive- and negative-sequence impedances. When only zero-sequence current flows in a transmission line, the current in each phase is identical. The current returns through the ground, through overhead ground wires, or through both. Because zero-sequence current is identical in each phase conductor (rather than equal only in magnitude and displaced in phase by 120 from other phase currents), the magnetic field due to zero-sequence current is very different from the magnetic field caused by either positive- or negative-sequence current. The difference in magnetic field results in the zero-sequence inductive reactance of overhead transmission lines being 2 to 3.5 times as large as the positive-sequence reactance. The ratio is toward the higher portion of the specified range for double-circuit lines and lines without ground wires.

Example 11.5. In Fig. 11.10 the terminal voltages at the left-hand and right-hand ends of the line are given by

$$V_{an} = 182.0 + j70.0 \text{ kV} \qquad V_{a'n'} = 154.0 + j28.0 \text{ kV}$$

$$V_{bn} = 72.24 - j32.62 \text{ kV} \qquad V_{b'n'} = 44.24 - j74.62 \text{ kV}$$

$$V_{cn} = -170.24 + j88.62 \text{ kV} \qquad V_{c'n'} = -198.24 + j46.62 \text{ kV}$$

The line impedances in ohms are

$$Z_{aa} = j60 \qquad Z_{ab} = j20 \qquad Z_{nn} = j80 \qquad Z_{an} = \text{[illegible]}$$

Determine the line currents I_a, I_b, and I_c using symmetrical components. Repeat the solution without using symmetrical components.

Solution. The sequence impedances have calculated values

$$Z_0 = Z_{aa} + 2Z_{ab} + 3Z_{nn} - 6Z_{an} = j60 + j40 + j240 - j180 = j160\ \Omega$$

$$Z_1 = Z_2 = Z_{aa} - Z_{ab} = j60 - j20 = j40\ \Omega$$

The sequence components of the voltage drops in the line are

$$\begin{bmatrix} V_{aa'}^{(0)} \\ V_{aa'}^{(1)} \\ V_{aa'}^{(2)} \end{bmatrix} = \mathbf{A}^{-1} \begin{bmatrix} V_{an} - V_{a'n'} \\ V_{bn} - V_{b'n'} \\ V_{cn} - V_{c'n'} \end{bmatrix} = \mathbf{A}^{-1} \begin{bmatrix} (182.0 - 154.0) + j(70.0 - 28.0) \\ (72.24 - 44.24) - j(32.62 - 74.62) \\ -(170.24 - 198.24) + j(88.62 - 46.62) \end{bmatrix}$$

$$= \mathbf{A}^{-1} \begin{bmatrix} 28.0 + j42.0 \\ 28.0 + j42.0 \\ 28.0 + j42.0 \end{bmatrix} = \begin{bmatrix} 28.0 + j42.0 \\ 0 \\ 0 \end{bmatrix} \text{kV}$$

Substituting in Eq. (11.59), we obtain

$$V_{aa'}^{(0)} = 28{,}000 + j42{,}000 = j160 I_a^{(0)}$$

$$V_{aa'}^{(1)} = 0 = j40 I_a^{(1)}$$

$$V_{aa'}^{(2)} = 0 = j40 I_a^{(2)}$$

from which we determine the symmetrical components of the currents in phase a,

$$I_a^{(0)} = 262.5 - j175 \text{ A} \qquad I_a^{(1)} = I_a^{(2)} = 0$$

The line currents are therefore

$$I_a = I_b = I_c = 262.5 - j175 \text{ A}$$

The self- and mutual impedances of Eq. (11.52) have values

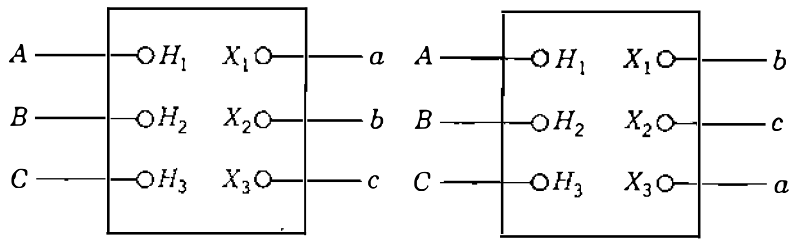
$$Z_s = Z_{aa} + Z_{nn} - 2Z_{an} = j60 + j80 - j60 = j80\ \Omega$$

$$Z_m = Z_{ab} + Z_{nn} - 2Z_{an} = j20 + j80 - j60 = j40\ \Omega$$

and so line currents can be calculated from Eq. (11.53) without symmetrical components as follows:

$$\begin{bmatrix} V_{aa'} \\ V_{bb'} \\ V_{cc'} \end{bmatrix} = \begin{bmatrix} 28 + j42 \\ 28 + j42 \\ 28 + j42 \end{bmatrix} \times 10^3 = \begin{bmatrix} j80 & j40 & j40 \\ j40 & j80 & j40 \\ j40 & j40 & j80 \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}$$

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} j80 & j40 & j40 \\ j40 & j80 & j40 \\ j40 & j40 & j80 \end{bmatrix}^{-1} \begin{bmatrix} 28 + j42 \\ 28 + j42 \\ 28 + j42 \end{bmatrix} \times 10^3 = \begin{bmatrix} 262.5 - j175 \\ 262.5 - j175 \\ 262.5 - j175 \end{bmatrix} \text{A}$$



(a) $V_A^{(1)}$ leads $V_a^{(1)}$ by 30° (b) $V_A^{(1)}$ leads $V_b^{(1)}$ by 30°

FIGURE 11.23
Labeling of lines connected to a three-phase Y-Δ transformer.

diagram of Fig. 11.21 and the connection diagram of Fig. 11.23(*a*) both satisfy the ANSI requirement; and because the connections of the phases to the transformer terminals H_1, H_2, H_3 – X_1, X_2, X_3 are respectively marked A, B, C – a, b, c as shown in those figures, we find that the positive-sequence voltage to neutral $V_{AN}^{(1)}$ leads the positive-sequence voltage to neutral $V_{an}^{(1)}$ by 30°.

It is not absolutely necessary, however, to label lines attached to the transformer terminals X_1, X_2, and X_3 by the letters a, b, and c, respectively, as we have done, since no standards have been adopted for such labeling. In fact, in calculations the designation of lines could be chosen as shown in Fig. 11.23(*b*), which shows the letters b, c, and a associated, respectively, with X_1, X_2, and X_3. If the scheme of Fig. 11.23(*b*) is preferred, it is necessary only to exchange b for a, c for b, and a for c in the wiring and phasor diagrams of Fig. 11.21, which would then show that $V_{an}^{(1)}$ leads $V_{AN}^{(1)}$ by 90° and that $V_{an}^{(2)}$ lags $V_{AN}^{(2)}$ by 90°. It is easy to show that similar statements also apply to the corresponding currents.

We shall continue to follow the labeling scheme of Fig. 11.23(*a*), and Eqs. (11.88) then become identical to the ANSI requirement. When problems involving unsymmetrical faults are solved, positive- and negative-sequence components are found separately and phase shift is taken into account, if necessary, by applying Eqs. (11.88). Computer programs can be written to incorporate the effects of phase shift.

A transformer in a three-phase circuit may consist of three individual single-phase units, or it may be a three-phase transformer. Although the zero-sequence series impedances of three-phase units may differ slightly from the positive- and negative-sequence values, it is customary to assume that series impedances of all sequences are equal regardless of the type of transformer. Table A.1 in the Appendix lists tranformer reactances. Reactance and impedance are almost equal for transformers of 1000 kVA or larger. For simplicity in our calculations we omit the shunt admittance which accounts for exciting current.

11.9 UNSYMMETRICAL SERIES IMPEDANCES

In the previous sections we have been concerned particularly with systems that are normally balanced. Let us look, however, at the equations of a three-phase

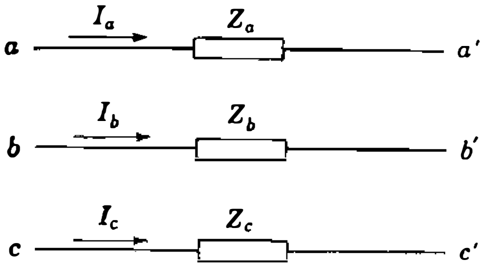


FIGURE 11.24
Portion of a three-phase system showing three unequal series impedances.

circuit when the series impedances are unequal. We shall reach a conclusion that is important in analysis by symmetrical components. Figure 11.24 shows the unsymmetrical part of a system with three unequal series impedances Z_a, Z_b, and Z_c. If we assume no mutual inductance (no coupling) among the three impedances, the voltage drops across the part of the system shown are given by the matrix equation

$$\begin{bmatrix} V_{aa'} \\ V_{bb'} \\ V_{cc'} \end{bmatrix} = \begin{bmatrix} Z_a & 0 & 0 \\ 0 & Z_b & 0 \\ 0 & 0 & Z_c \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} \tag{11.89}$$

and in terms of the symmetrical components of voltage and current

$$\mathbf{A} \begin{bmatrix} V_{aa'}^{(0)} \\ V_{aa'}^{(1)} \\ V_{aa'}^{(2)} \end{bmatrix} = \begin{bmatrix} Z_a & 0 & 0 \\ 0 & Z_b & 0 \\ 0 & 0 & Z_c \end{bmatrix} \mathbf{A} \begin{bmatrix} I_a^{(0)} \\ I_a^{(1)} \\ I_a^{(2)} \end{bmatrix} \tag{11.90}$$

where **A** is the matrix defined by Eq. (11.9). Premultiplying both sides of the equation by $\mathbf{A}^{-1}$ yields the matrix equation from which we obtain

$$\begin{aligned} V_{aa'}^{(0)} &= \tfrac{1}{3} I_a^{(0)}(Z_a + Z_b + Z_c) + \tfrac{1}{3} I_a^{(1)}(Z_a + a^2 Z_b + aZ_c) \\ &\quad + \tfrac{1}{3} I_a^{(2)}(Z_a + aZ_b + a^2 Z_c) \\ V_{aa'}^{(1)} &= \tfrac{1}{3} I_a^{(0)}(Z_a + aZ_b + a^2 Z_c) + \tfrac{1}{3} I_a^{(1)}(Z_a + Z_b + Z_c) \\ &\quad + \tfrac{1}{3} I_a^{(2)}(Z_a + a^2 Z_b + aZ_c) \\ V_{aa'}^{(2)} &= \tfrac{1}{3} I_a^{(0)}(Z_a + a^2 Z_b + aZ_c) + \tfrac{1}{3} I_a^{(1)}(Z_a + aZ_b + a^2 Z_c) \\ &\quad + \tfrac{1}{3} I_a^{(2)}(Z_a + Z_b + Z_c) \end{aligned} \tag{11.91}$$

If the impedances are made equal (that is, if $Z_a = Z_b = Z_c$), Eqs. (11.91) reduce to

$$V_{aa'}^{(0)} = I_a^{(0)} Z_a \qquad V_{aa'}^{(1)} = I_a^{(1)} Z_a \qquad V_{aa'}^{(2)} = I_a^{(2)} Z_a \tag{11.92}$$

If the impedances are unequal, however, Eqs. (11.91) show that the voltage drop of any one sequence is dependent on the currents of all three sequences. Thus, we conclude that the symmetrical components of unbalanced currents flowing in a *balanced* load or in *balanced* series impedances produce voltage drops of like sequence only. If asymmetric coupling (such as unequal mutual inductances) existed among the three impedances of Fig. 11.24, the square matrix of Eqs. (11.89) and (11.90) would contain off-diagonal elements and Eqs. (11.91) would have additional terms.

Although current in any conductor of a three-phase transmission line induces a voltage in the other phases, the way in which reactance is calculated eliminates consideration of coupling. The self-inductance calculated on the basis of complete transposition includes the effect of mutual reactance. The assumption of transposition yields equal series impedances. Thus, the component currents of any one sequence produce only voltage drops of like sequence in a transmission line; that is, positive-sequence currents produce positive-sequence voltage drops only. Likewise, negative-sequence currents produce negative-sequence voltage drops only and zero-sequence currents produce zero-sequence voltage drops only. Equations (11.91) apply to unbalanced Y loads because points a', b', and c' may be connected to form a neutral. We could study variations of these equations for special cases such as single-phase loads where $Z_b = Z_c = 0$, but we continue to confine our discussion to systems that are balanced before a fault occurs.

11.10 SEQUENCE NETWORKS

In the preceding sections of this chapter we have developed single-phase equivalent circuits in the form of zero-, positive-, and negative-sequence circuits for load impedances, transformers, transmission lines, and synchronous machines, all of which constitute the main parts of the three-phase power transmission network. Except for rotating machines, all parts of the network are static and without sources. Each individual part is assumed to be linear and three-phase symmetrical when connected in Y or Δ configuration. On the basis of these assumptions, we have found that:

- In any part of the network voltage drop caused by current of a certain sequence depends on only the impedance of that part of the network to current flow of that sequence.
- The impedances to positive- and negative-sequence currents, Z_1 and Z_2, are equal in any static circuit and may be considered approximately equal in synchronous machines under subtransient conditions.

- In any part of the network impedance to zero-sequence current, Z_0, is generally different from Z_1 and Z_2.
- Only positive-sequence circuits of rotating machines contain sources which are of positive-sequence voltages.
- Neutral is the reference for voltages in positive- and negative-sequence circuits, and such voltages to neutral are the same as voltages to ground if a physical connection of zero or other finite impedance exists between neutral and ground in the actual circuit.
- No positive- or negative-sequence currents flow between neutral points and ground.
- Impedances Z_n in the physical connections between neutral and ground are not included in positive- and negative-sequence circuits but are represented by impedances $3Z_n$ between the points for neutral and ground in the zero-sequence circuits only.

These characteristics of individual sequence circuits guide the construction of corresponding *sequence networks*. The object of obtaining the values of the sequence impedances of the various parts of a power system is to enable us to construct the sequence networks for the complete system. The network of a particular sequence—constructed by joining together all the corresponding sequence circuits of the separate parts—shows all the paths for the flow of current of that sequence in one phase of the actual system.

In a balanced three-phase system the currents flowing in the three phases under normal operating conditions constitute a symmetrical positive-sequence set. These positive-sequence currents cause voltage drops of the same sequence only. Because currents of only *one* sequence occurred in the preceding chapters, we considered them to flow in an independent *per-phase* network which combined the positive-sequence emfs of rotating machines and the impedances of other static circuits to positive-sequence currents only. That same per-phase equivalent network is now called the *positive-sequence network* in order to distinguish it from the networks of the other two sequences.

We have discussed the construction of impedance and admittance representations of some rather complex positive-sequence networks in earlier chapters. Generally, we have not included the phase shift associated with Δ-Y and Y-Δ transformers in positive-sequence networks since practical systems are designed with such phase shifts summing to zero around all loops. In detailed calculations, however, we must remember to *advance* all positive-sequence voltages and currents by 30° when *stepping up* from the low-voltage side to the high-voltage side of a Δ-Y or Y-Δ transformer.

The transition from a positive-sequence network to a *negative-sequence network* is simple. Three-phase synchronous generators and motors have internal voltages of positive sequence only because they are designed to generate balanced voltages. Since the positive- and negative-sequence impedances are the same in a static symmetrical system, conversion of a positive-sequence network to a negative-sequence network is accomplished by changing, if necessary, only the impedances that represent rotating machinery and by omitting the emfs. Electromotive forces are omitted on the assumption of balanced generated voltages and the absence of negative-sequence voltages induced from outside sources. Of course, in using the negative-sequence network for detailed calculations, we must also remember to *retard* the negative-sequence voltages and currents by 30° when *stepping up* from the low-voltage side to the high-voltage side of a Δ-Y or Y-Δ transformer.

Since all the neutral points of a symmetrical three-phase system are at the same potential when balanced three-phase currents are flowing, all the neutral points must be at the same potential for either positive- or negative-sequence currents. Therefore, the neutral of a symmetrical three-phase system is the logical reference potential for specifying positive- and negative-sequence voltage drops and is the reference node of the positive- and negative-sequence networks. Impedance connected between the neutral of a machine and ground is not a part of either the positive- or negative-sequence network because neither positive- nor negative-sequence current can flow in an impedance so connected.

Negative-sequence networks, like the positive-sequence networks of previous chapters, may contain the exact equivalent circuits of parts of the system or be simplified by omitting series resistance and shunt admittance.

Example 11.8. Draw the negative-sequence network for the system described in Example 6.1. Assume that the negative-sequence reactance of each machine is equal to its subtransient reactance. Omit resistance and phase shifts associated with the transformer connections.

Solution. Since all the negative-sequence reactances of the system are equal to the positive-sequence reactances, the negative-sequence network is identical to the positive-sequence network of Fig. 6.6 except for the omission of emfs from the negative-sequence network. The required network is drawn without transformer phase shifts in Fig. 11.25.

Zero-sequence equivalent circuits determined for the various separate parts of the system are readily combined to form the complete *zero-sequence network*. A three-phase system operates single phase insofar as the zero-

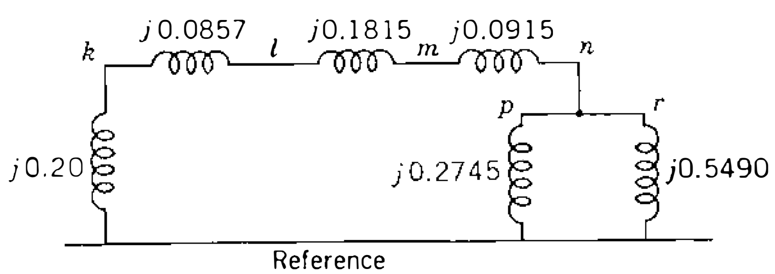


FIGURE 11.25
Negative-sequence network for Example 11.8.

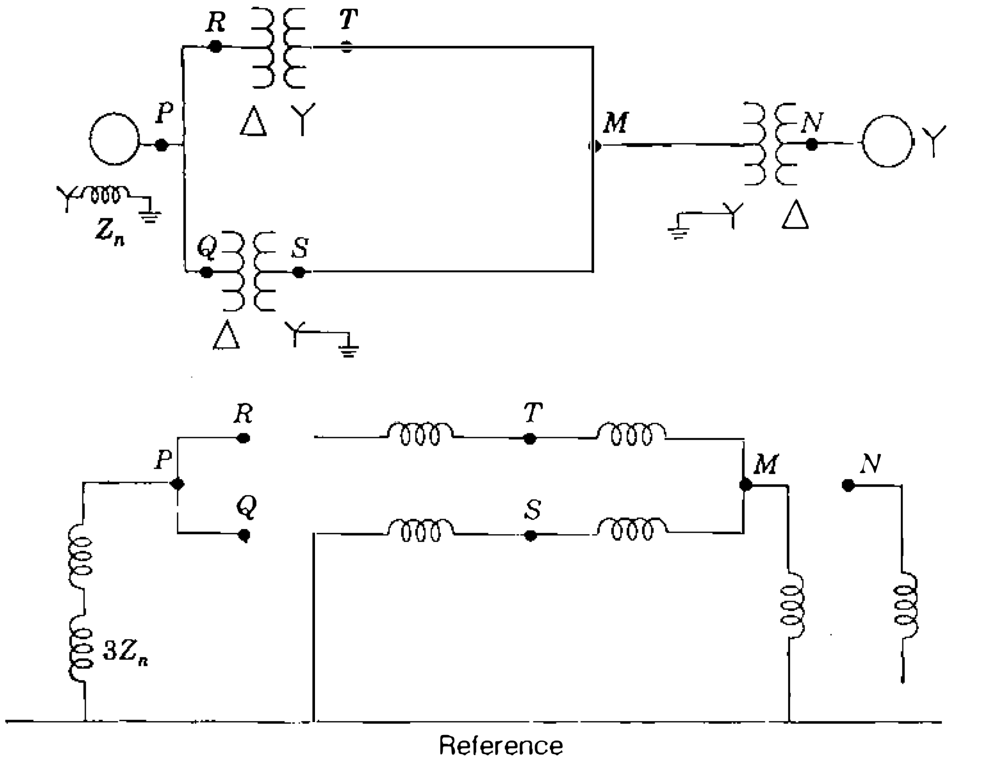


FIGURE 11.26
One-line diagram of a small power system and the corresponding zero-sequence network.

sequence currents are concerned, for the zero-sequence currents are the same in magnitude and phase at any point in all the phases of the system. Therefore, zero-sequence currents will flow only if a return path exists through which a completed circuit is provided. The reference for zero-sequence voltages is the potential of the ground at the point in the system at which any particular voltage is specified. Since zero-sequence currents may be flowing in the ground, the ground is not necessarily at the same potential at all points and the reference node of the zero-sequence network does not represent a ground of uniform potential. We have already discussed the fact that the impedance of the ground and ground wires is included in the zero-sequence impedance of the transmission line, and the return circuit of the zero-sequence network is a conductor of zero impedance, which is the reference node of the system. It is because the impedance of the ground is included in the zero-sequence impedance that voltages measured to the reference node of the zero-sequence network give the correct voltage to equivalent ideal ground. Figures 11.26 and 11.27 show one-line diagrams of two small power systems and their corresponding zero-sequence networks simplified by omitting resistances and shunt admittances.

The analysis of an unsymmetrical fault on a symmetrical system consists in finding the symmetrical components of the unbalanced currents that are flowing. Therefore, to calculate the effect of a fault by the method of symmetrical

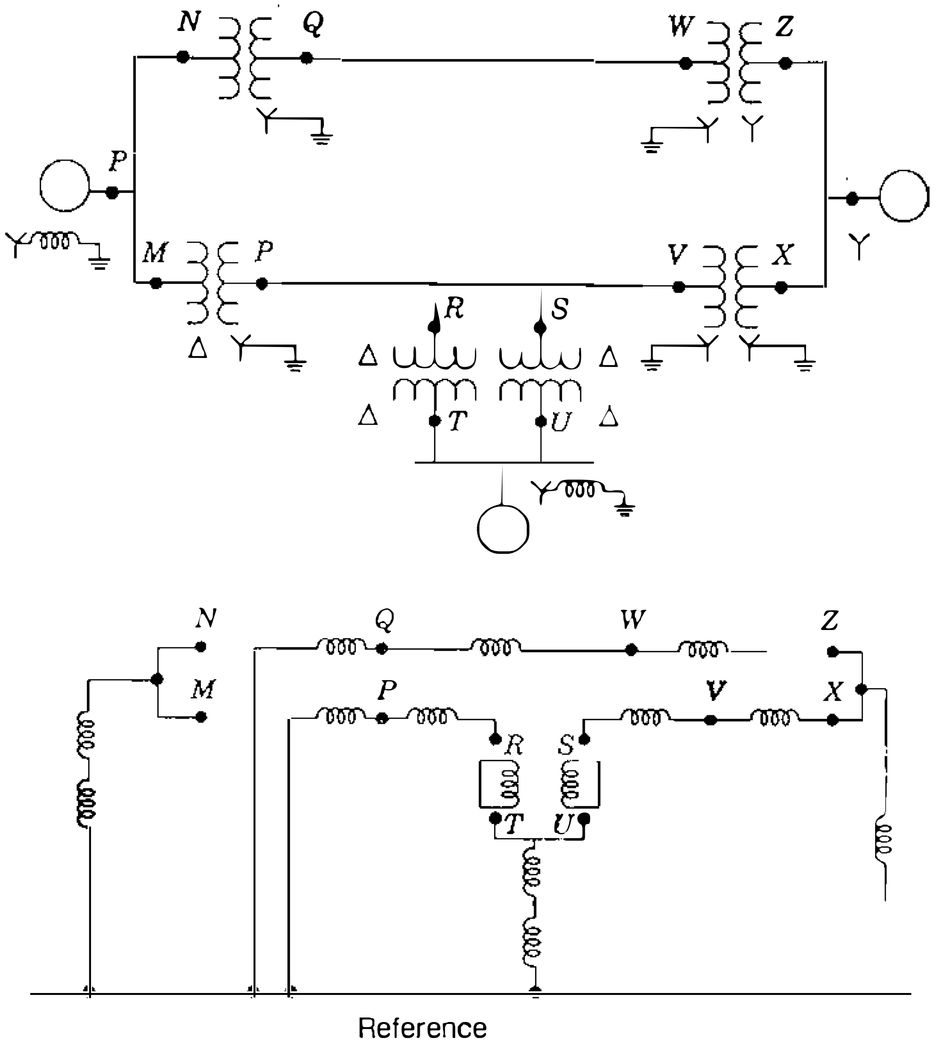


FIGURE 11.27
One-line diagram of a small power system and the corresponding zero-sequence network.

components, it is essential to determine the sequence impedances and to combine them to form the sequence networks. The sequence networks carrying the symmetrical-component currents $I_a^{(0)}$, $I_a^{(1)}$, and $I_a^{(2)}$ are then interconnected to represent various unbalanced fault conditions, as described in Chap. 12.

Example 11.9. Draw the zero-sequence network for the system described in Example 6.1. Assume zero-sequence reactances for the generator and motors of 0.05 per unit. A current-limiting reactor of 0.4 Ω is in each of the neutrals of the generator and the larger motor. The zero-sequence reactance of the transmission line is 1.5 Ω/km.

Solution. The zero-sequence leakage reactance of transformers is equal to the positive-sequence reactance. So, for the transformers $X_0 = 0.0857$ per unit and 0.0915 per unit, as in Example 6.1. Zero-sequence reactances of the generator and motors are

$$\text{Generator:} \quad X_0 = 0.05 \text{ per unit}$$

$$\text{Motor 1:} \quad X_0 = 0.05\left(\frac{300}{200}\right)\left(\frac{13.2}{13.8}\right)^2 = 0.0686 \text{ per unit}$$

$$\text{Motor 2:} \quad X_0 = 0.05\left(\frac{300}{100}\right)\left(\frac{13.2}{13.8}\right)^2 = 0.1372 \text{ per unit}$$

In the generator circuit

$$\text{Base } Z = \frac{(20)^2}{300} = 1.333\ \Omega$$

and in the motor circuit

$$\text{Base } Z = \frac{(13.8)^2}{300} = 0.635\ \Omega$$

In the impedance network for the generator

$$3Z_n = 3\left(\frac{0.4}{1.333}\right) = 0.900 \text{ per unit}$$

and for the motor

$$3Z_n = 3\left(\frac{0.4}{0.635}\right) = 1.890 \text{ per unit}$$

For the transmission line

$$Z_0 = \frac{1.5 \times 64}{176.3} = 0.5445 \text{ per unit}$$

The zero-sequence network is shown in Fig. 11.28.

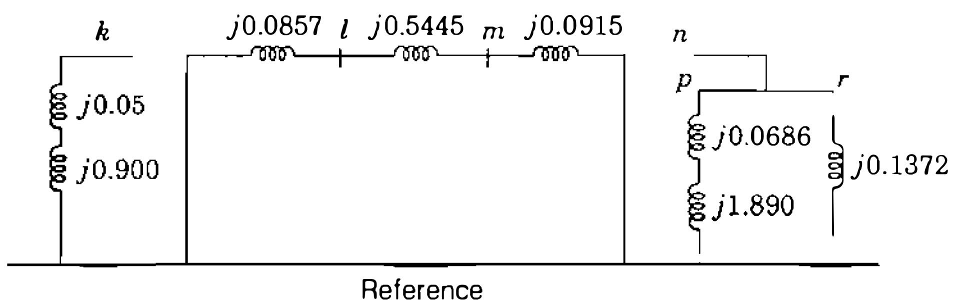


FIGURE 11.28
Zero-sequence network for Example 11.9.

11.11 SUMMARY

Unbalanced voltages and currents can be resolved into their symmetrical components. Problems are solved by treating each set of components separately and superimposing the results.

In balanced networks having strictly symmetrical coupling between phases the currents of one phase sequence induce voltage drops of like sequence only. Impedances of circuit elements to currents of different sequences are not necessarily equal.

A knowledge of the positive-sequence network is necessary for power-flow studies, fault calculations, and stability studies. If the fault calculations or stability studies involve unsymmetrical faults on otherwise symmetrical systems, the negative- and zero-sequence networks are also needed. Synthesis of the zero-sequence network requires particular care because the zero-sequence network may differ from the others considerably.

PROBLEMS

11.1. If $V_{an}^{(1)} = 50\angle 0°$, $V_{an}^{(2)} = 20\angle 90°$, and $V_{an}^{(0)} = 10\angle 180°$ V, determine analytically the voltages to neutral V_{an}, V_{bn}, and V_{cn}, and also show graphically the sum of the given symmetrical components which determine the line-to-neutral voltages.

11.2. When a generator has terminal a open and the other two terminals are connected to each other with a short circuit from this connection to ground, typical values for the symmetrical components of current in phase a are $I_a^{(1)} = 600\angle -90°$, $I_a^{(2)} = 250\angle 90°$, and $I_a^{(0)} = 350\angle 90°$ A. Find the current into the ground and the current in each phase of the generator.

11.3. Determine the symmetrical components of the three currents $I_a = 10\angle 0°$, $I_b = 10\angle 230°$, and $I_c = 10\angle 130°$ A.

11.4. The currents flowing in the lines toward a balanced load connected in Δ are $I_a = 100\angle 0°$, $I_b = 141.4\angle 225°$, and $I_c = 100\angle 90°$. Find the symmetrical components of the given line currents and draw phasor diagrams of the positive- and negative-sequence line and phase currents. What is I_{ab} in amperes?

11.5. The voltages at the terminals of a balanced load consisting of three 10-Ω resistors connected in Y are $V_{ab} = 100\angle 0°$, $V_{bc} = 80.8\angle -121.44°$, and $V_{ca} = 90\angle 130°$ V. Assuming that there is no connection to the neutral of the load, find the line currents from the symmetrical components of the given line voltages.

11.6. Find the power expended in the three 10-Ω resistors of Prob. 11.5 from the symmetrical components of currents and voltages. Check the answer.

11.7. If there is impedance in the neutral connection to ground of a Y-connected load, then show that the voltages V_a, V_b, and V_c of Eq. (11.26) must be interpreted as voltages with respect to ground.

11.8. A balanced three-phase load consists of Δ-connected impedances Z_Δ in parallel with solidly grounded Y-connected impedances Z_Y.

(*a*) Express the currents I_a, I_b, and I_c flowing in the lines from the supply source toward the load in terms of the source voltages V_a, V_b, and V_c.

(*b*) Transform the expressions of part (*a*) into their symmetrical component equivalents, and thus express $I_a^{(0)}$, $I_a^{(1)}$, and $I_a^{(2)}$ in terms of $V_a^{(0)}$, $V_a^{(1)}$, and $V_a^{(2)}$.

(*c*) Hence, draw the sequence circuit for the combined load.

11.9. The Y-connected impedances in parallel with the Δ-connected impedances Z_Δ of Prob. 11.8 are now grounded through an impedance Z_g.

(*a*) Express the currents I_a, I_b, and I_c flowing in the lines from the supply source toward the load in terms of the source voltages V_a, V_b, and V_c and the voltage V_n of the neutral point.

(*b*) Expressing V_n in terms of $I_a^{(0)}$, $I_a^{(1)}$, $I_a^{(2)}$, and Z_g, find the equations for these currents in terms of $V_a^{(0)}$, $V_a^{(1)}$, and $V_a^{(2)}$.

(*c*) Hence, draw the sequence circuit for the combined load.

11.10. Suppose that the line-to-neutral voltages at the sending end of the line described in Example 11.5 can be maintained constant at 200-kV and that a single-phase inductive load of 420 Ω is connected between phase *a* and neutral at the receiving end.

(*a*) Use Eqs. (11.51) to express numerically the receiving-end sequence voltages $V_{a'n'}^{(0)}$, $V_{a'n'}^{(1)}$, and $V_{a'n'}^{(2)}$ in terms of the load current I_L and the sequence impedances Z_0, Z_1, and Z_2 of the line.

(*b*) Hence, determine the line current I_L in amperes.

(*c*) Determine the open-circuit voltages to neutral of phases *b* and *c* at the receiving end.

(*d*) Verify your answer to part (*c*) without using symmetrical components.

11.11. Solve Prob. 11.10 if the same 420-Ω inductive load is connected between phases *a* and *b* at the receiving end. In part (*c*) find the open-circuit voltage of phase *c* only.

11.12. A Y-connected synchronous generator has sequence reactances $X_0 = 0.09$, $X_1 = 0.22$, and $X_2 = 0.36$, all in per unit. The neutral point of the machine is grounded through a reactance of 0.09 per unit. The machine is running on no load with rated terminal voltage when it suffers an unbalanced fault. The fault currents out of the machine are $I_a = 0$, $I_b = 3.75\angle 150°$, and $I_c = 3.75\angle 30°$, all in per unit with respect to phase *a* line-to-neutral voltage. Determine

(*a*) The terminal voltages in each phase of the machine with respect to ground,

(*b*) The voltage of the neutral point of the machine with respect to ground, and

(*c*) The nature (type) of the fault from the results of part (*a*).

11.13. Solve Prob. 11.12 if the fault currents in per unit are $I_a = 0$, $I_b = -2.986\angle 0°$, and $I_c = 2.986\angle 0°$.

11.14. Assume that the currents specified in Prob. 11.4 are flowing toward a load from lines connected to the Y side of a Δ-Y transformer rated 10 MVA, 13.2Δ/66Y kV. Determine the currents flowing in the lines on the Δ side by converting the symmetrical components of the currents to per unit on the base of the transformer rating and by shifting the components according to Eq. (11.88). Check the results by computing the currents in each phase of the Δ windings in amperes directly from the currents on the Y side by multiplying by the turns ratio of the windings. Complete the check by computing the line currents from the phase currents on the Δ side.

11.15. Three single-phase transformers are connected as shown in Fig. 11.29 to form a Y-Δ transformer. The high-voltage windings are Y-connected with polarity marks as indicated. Magnetically coupled windings are drawn in parallel directions. Determine the correct placement of polarity marks on the low-voltage windings. Identify the numbered terminals on the low-voltage side (*a*) with the letters *a*, *b*, and *c*, where $I_A^{(1)}$ leads $I_a^{(1)}$ by 30, and (*b*) with the letters *a'*, *b'*, and *c'* so that $I_{a'}^{(1)}$ is 90 out of phase with $I_A^{(1)}$.

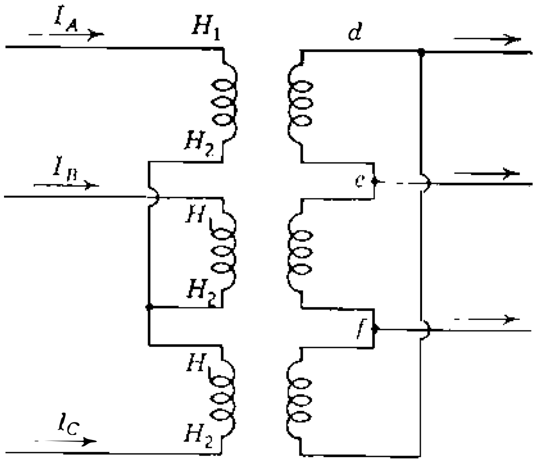


FIGURE 11.29
Circuit for Prob. 11.15.

11.16. Balanced three-phase voltages of 100 V line to line are applied to a Y-connected load consisting of three resistors. The neutral of the load is not grounded. The resistance in phase *a* is 10 Ω, in phase *b* is 20 Ω, and in phase *c* is 30 Ω. Select voltage to neutral of the three-phase line as reference and determine the current in phase *a* and the voltage V_{an}.

11.17. Draw the negative- and zero-sequence impedance networks for the power system of Prob. 3.12. Mark the values of all reactances in per unit on a base of 50 MVA, 13.8 kV in the circuit of generator 1. Letter the networks to correspond to the single-line diagram. The neutrals of generators 1 and 3 are connected to ground through current-limiting reactors having a reactance of 5%, each on the base of the machine to which it is connected. Each generator has negative- and zero-sequence reactances of 20 and 5%, respectively, on its own rating as base. The zero-sequence reactance of the transmission line is 210 Ω from *B* to *C* and 250 Ω from *C* to *E*.

11.18. Draw the negative- and zero-sequence impedance networks for the power system of Prob. 3.13. Choose a base of 50 MVA, 138 kV in the 40-Ω transmission line and mark all reactances in per unit. The negative-sequence reactance of each synchronous machine is equal to its subtransient reactance. The zero-sequence reactance of each machine is 8% based on its own rating. The neutrals of the machines are connected to ground through current-limiting reactors having a reactance of 5%, each on the base of the machine to which it is connected. Assume that the zero-sequence reactances of the transmission lines are 300% of their positive-sequence reactances.

11.19. Determine the zero-sequence Thévenin impedance seen at bus © of the system described in Prob. 11.17 if transformer T_3 has (*a*) one ungrounded and one solidly grounded neutral, as shown in Fig. 3.23, and (*b*) both neutrals are solidly grounded.

13

Symmetrical Components and Fault Calculations

INTRODUCTION

In 1918, Dr. C.L. Fortescue presented a paper entitled "Method of Symmetrical Coordinates Applied to Solution of Polyphase Networks" at AIEE in which he proved that "a system of n vectors or quantities may be resolved, when n is prime, into n different symmetrical groups or systems, one of which consists of n equal vectors and the remaining $(n - 1)$ systems consist of n equi-spaced vectors which with the first mentioned group of equal vectors forms an equal number of symmetrical n-phase systems".

The method of symmetrical components is a general one applicable to any polyphase system.

Because of the widespread use of 3-phase systems and the greater familiarity which electrical engineers have with them, symmetrical component equations will be developed for 3-phase systems.

13.1 3-PHASE SYSTEMS

Any three coplanar vectors V_a, V_b and V_c can be expressed in terms of three new vectors V_1, V_2 and V_3 by three simultaneous linear equations with constant coefficients. Thus

$$V_a = a_{11}V_1 + a_{12}V_2 + a_{13}V_3 \qquad (13.1)$$

$$V_b = a_{21}V_1 + a_{22}V_2 + a_{23}V_3 \qquad (13.2)$$

$$V_c = a_{31}V_1 + a_{32}V_2 + a_{33}V_3 \qquad (13.3)$$

Each of the original vectors has been replaced by a set of three vectors making a total of nine vectors. This has been done to simplify the calculations and to have better understanding of the problem. With this in mind, two conditions should be satisfied in selecting systems of components to replace 3-phase current and voltage vectors:

1. Calculations should be simplified by the use of the chosen systems of components. This is possible only if the impedances (or admittances) associated with the components of current (or voltage) can be obtained readily by calculation or test.

2. The system of components chosen should have physical significance and be an aid in determining power system performance.

According the Fortescue's theorem, the three unbalanced vectors V_a, V_b and V_c can be replaced by a set of three balanced systems of vectors. Therefore, the solution of equations (13.1)–(13.3) is unique. A balanced system of three vectors is one in which the vectors are equal in magnitude and are equi-spaced. The three symmetrical component vectors replacing V_a, V_b and V_c are:

1. Positive sequence component which has three vectors of equal magnitude but displaced in phase from each other by 120° and has the same phase sequence as the original vectors.

2. Negative sequence component which has three vectors of equal magnitude but displaced in phase from each other by 120° and has the phase sequence opposite to the original vectors.

3. Zero sequence component which has three vectors of equal magnitude and also are in phase with each other.

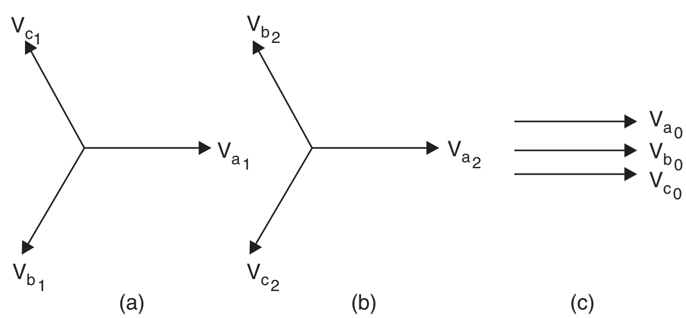


Fig. 13.1 (*a*) Positive sequence component; (*b*) Negative sequence component; (*c*) Zero sequence component.

The components have been shown in Fig. 13.1. The voltage vectors have been designated as V_a, V_b and V_c and the phase sequence is assumed here as a, b, c. The subscripts 1, 2 and 0 are being used to represent positive, negative and zero sequence quantities respectively.

13.2 SIGNIFICANCE OF POSITIVE, NEGATIVE AND ZERO SEQUENCE COMPONENTS

By a positive sequence system of vectors is meant the vectors are equal in magnitude and 120° apart in phase, in which the time order of arrival of the phase vectors at a fixed axis of reference corresponds to the generated voltages. This really means that if a set of positive sequence voltages is applied to the stator winding of the alternator, the direction of rotation of the stator field is the same as the rotor or alternatively if the direction of rotation of the stator field is the same as that of the rotor, the set of voltages are positive sequence voltages. On the contrary if the direction of rotation of the stator field is opposite to that of the rotor, the set of voltages are negative sequence voltages. The zero sequence voltages are single phase voltages and, therefore, they give rise to an alternating field in space. Since the 3-phase windings are 120° apart in space, at any particular instant the three vector fields due to the three phases are 120° apart and, therefore, assuming complete symmetry of the windings, the net flux in the air gap will be zero.

From Fig. 13.1, the following relations between the original unbalanced vectors and their corresponding symmetrical components, can be written:

$$V_a = V_{a_1} + V_{a_2} + V_{a_0} \quad (13.4)$$

$$V_b = V_{b_1} + V_{b_2} + V_{b_0} \quad (13.5)$$

$$V_c = V_{c_1} + V_{c_2} + V_{c_0} \quad (13.6)$$

Assuming phase a as the reference as shown in Fig. 13.1 the following relations between the symmetrical components of phases b and c in terms of phase a can be written. Here use is made of the operator λ which has a magnitude of unity and rotation through 120°, *i.e.*, when any vector is multiplied by λ, the vector magnitude remains same but is rotated anticlockwise through 120°. Thus

$$\lambda = 1\angle 120°$$

In the complex form $\lambda = \cos 120° + j \sin 120°$

$$= -0.5 + j0.866$$

Similarly $\lambda^2 = -0.5 - j0.866$

$$\lambda^3 = 1.0 = 1\angle 360°$$

or $\lambda^3 - 1 = 0$

or $(\lambda - 1)(\lambda^2 + \lambda + 1) = 0$

Since $\lambda \neq 1$ as λ is a complex quantity as defined above,

$\therefore$ $\lambda^2 + \lambda + 1 = 0$

In fact λ is a number which when doubly squared remains λ itself, *i.e.*, $\lambda^4 = \lambda$.

So the important relations that will be frequently required in power system analysis are

$$\lambda = -0.5 + j0.866 = 1.0\angle 120°$$

$$\lambda^2 = -0.5 - j0.866 = 1.0\angle -120°$$

$$\lambda^3 = 1.0\angle 0°$$

$$\lambda^4 = \lambda$$

$$\lambda^2 + \lambda + 1 = 0$$

Now we go back to deriving relations between the symmetrical components of phases b and c in terms of the symmetrical components of phase a.

From Fig. 13.1, $V_{b_1} = \lambda^2 V_{a_1}$

This means in order to express V_{b_1} in terms of V_{a_1}, V_{a_1} should be rotated anti-clockwise through 240°.

Similarly $V_{c_1} = \lambda V_{a_1}$

For negative sequence vectors

$$V_{b_1} = \lambda V_{a_2}, \; V_{c_2} = \lambda^2 V_{a_2}$$

For zero sequence vectors

$$V_{b_0} = V_{a_0} = V_{c_0}$$

Substituting these relations in equations (13.4)–(13.6),

$$V_a = V_{a_1} + V_{a_2} + V_{c_0} \quad (13.7)$$

$$V_b = \lambda^2 V_{a_1} + \lambda V_{a_2} + V_{a_0} \quad (13.8)$$

$$V_c = \lambda V_{a_1} + \lambda^2 V_{a_2} + V_{a_0} \quad (13.9)$$

Compare equations (13.1)–(13.3) with equations (13.7)–(13.9),

$$a_{11} = a_{12} = a_{13} = 1$$

$$a_{21} = \lambda^2, a_{22} = \lambda, a_{23} = 1$$

$$a_{31} = \lambda, a_{32} = \lambda^2, a_{33} = 1$$

The coefficients have been uniquely determined for the 3-phase systems. Equations (13.7)–(13.9) express the phase voltages V_a, V_b and V_c in terms of the symmetrical components of phase a *i.e.*, in case V_{a_1}, V_{a_2} and V_{a_0} are known, the phase voltages V_a, V_b and V_c can be calculated.

Similar relations between the phase currents in terms of the symmetrical components of currents taking phase a as reference hold good and are given below:

$$I_a = I_{a_1} + I_{a_2} + I_{a_0} \quad (13.7a)$$

$$I_b = \lambda^2 I_{a_1} + \lambda I_{a_2} + I_{a_0} \quad (13.8a)$$

$$I_c = \lambda I_{a_1} + \lambda^2 I_{a_2} + I_{a_0} \quad (13.9a)$$

Normally the unbalanced phase voltages and currents are known in a system; it is required to find out the symmetrical components. The procedure is as follows:

The problem is: given V_a, V_b, V_c, find out V_{a_1}, V_{a_2} and V_{a_0}. To find out positive sequence component V_{a_1}, multiply equations (13.7), (13.8) and (13.9) by 1, λ and λ^2 respectively and adding them up, it gives

$$V_a + \lambda V_b + \lambda^2 V_c = V_{a_1}(1 + \lambda^3 + \lambda^3) + V_{a_2}(1 + \lambda^2 + \lambda^4) + V_{a_0}(1 + \lambda + \lambda^2)$$

$$= 3V_{a_1} + V_{a_2}(1 + \lambda^2 + \lambda) + 0$$

$$= 3V_{a_1}$$

Since $1 + \lambda + \lambda^2 = 0$

$$\therefore \quad V_{a_1} = \tfrac{1}{3}(V_a + \lambda V_b + \lambda^2 V_c)$$

For negative sequence component V_{a_2} multiplying equations (13.7), (13.8) and (13.9) by 1, λ^2 and λ respectively and adding,

$$V_a + \lambda^2 V_b + \lambda V_c = V_{a_1}(1 + \lambda^4 + \lambda^2) + V_{a_2}(1 + \lambda^3 + \lambda^3) + V_{a_0}(1 + \lambda^2 + \lambda)$$

$$= 3V_{a_2}$$

$$\therefore \quad V_{a_2} = \tfrac{1}{3}(V_a + \lambda^2 V_b + \lambda V_c)$$

For zero sequence component V_{a_0}, add equations (13.7), (13.8) and (13.9)

$$V_a + V_b + V_c = V_{a_1}(1 + \lambda^2 + \lambda) + V_{a_2}(1 + \lambda + \lambda^2) + 3V_{a_0}$$

or
$$V_{a_0} = \tfrac{1}{3}(V_a + V_b + V_c)$$

Rewriting these equations,

$$V_{a_1} = \tfrac{1}{3}(V_a + \lambda V_b + \lambda^2 V_c) \quad (13.10)$$

$$V_{a_2} = \tfrac{1}{3}(V_a + \lambda^2 V_b + \lambda V_c) \quad (13.11)$$

$$V_{a_0} = \tfrac{1}{3}(V_a + V_b + V_c) \quad (13.12)$$

Similarly these relations for currents are given as

$$I_{a_1} = \tfrac{1}{3}(I_a + \lambda I_b + \lambda^2 I_c)$$

$$I_{a_2} = \tfrac{1}{3}(I_a + \lambda^2 I_b + \lambda I_c)$$

$$I_{a_0} = \tfrac{1}{3}(I_a + I_b + I_c) \quad (13.13)$$

In the equations above V_a, V_b and V_c may be the line to ground voltages, line to neutral voltages, line to line voltages at a point in the network or they may be the generated or induced voltages, in fact any set of three voltages revolving at the same rate which may exist in the 3-phase system. Similarly, the three currents could be, phase currents, line currents, the currents flowing into a fault from the line conductors etc.

Example 13.1: The line-to-ground voltages on the high voltage side of a step-up transformer are 100 kV, 33 kV and 38 kV on phases *a*, *b* and *c* respectively. The voltage of phase *a* leads that of phase *b* by 100° and lags that of phase *c* by 176.5°. Determine analytically the symmetrical components of voltage

$$V_a = 100\angle 0°$$
$$V_b = 33\angle -100°$$
$$V_c = 38\angle 176.5°.$$

Solution:
$$V_{a_1} = \tfrac{1}{3}(V_a + \lambda V_b + \lambda^2 V_c)$$
$$= \tfrac{1}{3}[100\angle 0° + 33\angle -100° \,.\, \angle 120° + 38\angle 176.5°\angle -120°]$$
$$= \tfrac{1}{3}[100 + j0.0 + 33\angle 20° + 38\angle 56.5°]$$
$$= \tfrac{1}{3}[151.97 + j42.97] = 50.65 + j14.32. \quad \textbf{Ans.}$$

$$V_{a_2} = \tfrac{1}{3}[V_a + \lambda^2 V_b + \lambda V_c]$$
$$= \tfrac{1}{3}[100 + j0.0 + 33\angle -220° + 38\angle 296.5°]$$
$$= (30.55 - j4.26). \quad \textbf{Ans.}$$

Similarly,

$$V_{c_0} = \tfrac{1}{3}(V_a + V_b + V_c)$$
$$= \tfrac{1}{3}[100 + j0.0 + 33\angle -100° + 38\angle 176.5°]$$
$$= \tfrac{1}{3}[56.37 - j30.18]$$
$$= 18.79 - j10.06. \quad \textbf{Ans.}$$

Example 13.2: The line currents in amperes in phases *a*, *b* and *c* respectively are 500 + *j*150, 100 – *j*600 and – 300 + *j*600 referred to the same reference vector. Find the symmetrical component of currents.

Solution: The line currents are

$$I_a = 500 + j150,\ I_b = 100 - j600 \text{ and } I_c = -300 + j600 \text{ amps}$$

$$I_{a_0} = \tfrac{1}{3}(I_a + I_b + I_c)$$
$$= \tfrac{1}{3}[500 + j150 + 100 - j600 - 300 + j600]$$
$$= 100 + j\,50 \text{ amps.} \quad \textbf{Ans.}$$

$$I_{a_1} = \tfrac{1}{3}[I_a + \lambda I_b + \lambda^2 I_c]$$
$$= \tfrac{1}{3}[500 + j150 + (-0.5 + j0.866)(100 - j600) + (-0.5 - j0.866)(-300 + j600)]$$
$$= \tfrac{1}{3}[1639 + j496.4] = 546.3 + j165.46 \text{ amps.} \quad \textbf{Ans.}$$

$$I_{a_2} = \tfrac{1}{3}[I_a + \lambda^2 I_b + \lambda I_c]$$
$$= \tfrac{1}{3}[500 + j150 + (-0.5 - j0.866)(100 - j600) + (-0.5 + j0.866)(-300 + j600)]$$
$$= \tfrac{1}{3}[146.3 - j65.46]$$
$$= 48.8 - j21.82 \text{ amps.} \quad \textbf{Ans.}$$

13.3 AVERAGE 3-PHASE POWER IN TERMS OF SYMMETRICAL COMPONENTS

The average power

$$P = V_a I_a \cos\phi_a + V_b I_b \cos\phi_b + V_c I_c \cos\phi_c \tag{13.14}$$
$$= V_a \cdot I_a + V_b \cdot I_b + V_c \cdot I_c$$
$$= (V_{a_1} + V_{a_2} + V_{a_0}) \cdot (I_{a_1} + I_{a_2} + I_{a_0})$$
$$+ (\lambda^2 V_{a_1} + \lambda V_{a_2} + V_{a_0}) \cdot (\lambda^2 I_{a_1} + \lambda I_{a_2} + I_{a_0})$$
$$+ (\lambda V_{a_1} + \lambda^2 V_{a_2} + V_{a_0}) \cdot (\lambda I_{a_1} + \lambda^2 I_{a_2} + I_{a_0})$$

Taking first term on the r.h.s.,

$$(V_{a_1} + V_{a_2} + V_{a_0}) \cdot (I_{a_1} + I_{a_2} + I_{a_0})$$
$$= V_{a_1} \cdot I_{a_1} + V_{a_2} \cdot I_{a_2} + V_{a_0} \cdot I_{a_0} + V_{a_1} \cdot I_{a_2} + V_{a_1} \cdot I_{a_0} + V_{a_2} \cdot I_{a_1} + V_{a_2} \cdot I_{a_0} + V_{a_0} \cdot I_{a_1} + V_{a_0} \cdot I_{a_2}$$

Expanding second term on the r.h.s.,

$$(\lambda^2 V_{a_1} + \lambda V_{a_2} + V_{a_0}) \cdot (\lambda^2 I_{a_1} + \lambda I_{a_2} + I_{a_0})$$
$$= \lambda^2 V_{a_1} \cdot \lambda^2 I_{a_1} + \lambda^2 V_{a_1} \cdot \lambda I_{a_2} + \lambda^2 V_{a_1} \cdot I_{a_0} + \lambda V_{a_2} \cdot \lambda^2 I_{a_1} + \lambda V_{a_2} \cdot \lambda I_{a_2} + \lambda V_{a_2} \cdot I_{a_0} + V_{a_0} \cdot \lambda^2 I_{a_1} + V_{a_0} \cdot \lambda I_{a_2} + V_{a_0} \cdot I_{a_0}$$

Now the dot product of two vectors does not change when both are rotated through the same angle.

For example, $\lambda^2 V_{a_1} \cdot \lambda^2 I_{a_1} = V_{a_1} \cdot I_{a_1}$

$$\lambda^2 V_{a_1} \cdot \lambda I_{a_2} = \lambda V_{a_1} \cdot I_{a_2}$$

The addition of the terms after expanding and rearranging,

$$\begin{aligned} P &= 3V_{a_0} \cdot I_{a_0} + 3V_{a_2} \cdot I_{a_2} + 3V_{a_1} \cdot I_{a_1} + V_{a_1} \cdot I_{a_2}(1+\lambda+\lambda^2) \\ &\quad + V_{a_1} \cdot I_{a_0}(1+\lambda+\lambda^2) + V_{a_2} \cdot I_{a_1}(1+\lambda+\lambda^2) + V_{a_2} \cdot I_{a_0}(1+\lambda+\lambda^2) \\ &\quad + V_{a_0} \cdot I_{a_1}(1+\lambda+\lambda^2) + V_{a_0} \cdot I_{a_2}(1+\lambda+\lambda^2) \\ &= 3(V_{a_1} \cdot I_{a_1} + V_{a_2} \cdot I_{a_2} + V_{a_0} \cdot I_{a_0}) \\ &= 3[|V_{a_1}||I_{a_1}|\cos\theta_1 + |V_{a_2}||I_{a_2}|\cos\theta_2 \\ &\qquad + |V_{a_0}||I_{a_0}|\cos\theta_0] \end{aligned} \tag{13.15}$$

The same power expression can be very easily derived using matrix manipulations.

$$P + jQ = V_a I_a^* + V_b I_b^* + V_c I_c^*$$

$$= [V_a \quad V_b \quad V_c] \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}^* = \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix}^T \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}^*$$

Since from equations (13.7), (13.8) and (13.9),

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \lambda^2 & \lambda \\ 1 & \lambda & \lambda^2 \end{bmatrix} \begin{bmatrix} V_{a_0} \\ V_{a_1} \\ V_{a_2} \end{bmatrix} = AV$$

and

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix}^T = (AV)^T = V^T A^T$$

$$\therefore \qquad P + jQ = \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix}^T \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}^* = [V_{a_0} \quad V_{a_1} \quad V_{a_2}] \begin{bmatrix} 1 & 1 & 1 \\ 1 & \lambda^2 & \lambda \\ 1 & \lambda & \lambda^2 \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}^*$$

Now substituting for the phase currents the corresponding symmetrical components, noting that λ and λ^2 are conjugate,

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}^* = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \lambda^2 & \lambda \\ 1 & \lambda & \lambda^2 \end{bmatrix}^* \begin{bmatrix} I_{a_0} \\ I_{a_1} \\ I_{a_2} \end{bmatrix}^* = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \lambda & \lambda^2 \\ 1 & \lambda^2 & \lambda \end{bmatrix} \begin{bmatrix} I_{a_0} \\ I_{a_1} \\ I_{a_2} \end{bmatrix}^*$$

$$\therefore \qquad P + jQ = [V_{a_0} \quad V_{a_1} \quad V_{a_2}] \begin{bmatrix} 1 & 1 & 1 \\ 1 & \lambda^2 & \lambda \\ 1 & \lambda & \lambda^2 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \lambda & \lambda^2 \\ 1 & \lambda^2 & \lambda \end{bmatrix} \begin{bmatrix} I_{a_0} \\ I_{a_1} \\ I_{a_2} \end{bmatrix}^*$$

$$= [V_{a_0} \quad V_{a_1} \quad V_{a_2}] \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} I_{a_0} \\ I_{a_1} \\ I_{a_2} \end{bmatrix}^*$$

$$= 3[V_{a_0} \quad V_{a_1} \quad V_{a_2}] \begin{bmatrix} I_{a_0} \\ I_{a_1} \\ I_{a_2} \end{bmatrix}^*$$

$$= 3[V_{a_0} I_{a_0}^* + V_{a_1} I_{a_1}^* + V_{a_2} I_{a_2}^*]$$

$$\therefore \quad P = 3[\,|V_{a_0}|\,|I_{a_0}| \cos \theta_0 + |V_{a_1}|\,|I_{a_1}| \cos \theta_1 + |V_{a_2}|\,|I_{a_2}| \cos \theta_2].$$

13.4 SEQUENCE IMPEDANCES

So far we have discussed the symmetrical components for the currents, voltages and power. Let us now study something about the sequence impedances of the system. The sequence impedances of an equipment or a component of power system are the positive, negative and zero sequence impedances. They are defined as follows:

The positive sequence impedance of an equipment is the impedance offered by the equipment to the flow of positive sequence currents. Similarly, the negative sequence or zero sequence impedance of the equipment is the impedance offered by the equipment to the flow of corresponding sequence current. The significance of the positive, negative and zero sequence currents has already been discussed in this chapter. For a 3-phase, symmetrical static circuit without internal voltages like transformers and transmission lines, the impedances to the currents of any sequence are the same in the three phases; also the currents of a particular sequence will produce drop of the same sequence or a voltage of a particular sequence will produce current of the same sequence only, which means there is no mutual coupling between the sequence networks. Since for a static device, the sequence has no significance, the positive and negative sequence impedances are equal; the zero sequence impedance which includes the impedance of the return path through the ground, in the general case, is different from the positive and negative sequence impedance. In a symmetrical rotating machine the impedances met by armature currents of a given sequence are equal in the three phases. Since by definition the inductance, which forms a part of impedance, is the flux linkages per ampere, it will depend upon the phase order of the sequence current relative to the direction of rotation of the rotor; positive, negative and zero sequence impedances are unequal in the general case. In fact for a rotating machine, the positive sequence impedance varies, having minimum value immediately following the fault and then increases with time until steady state conditions are reached when the positive sequence impedance corresponds to the synchronous impedance. The variation of the positive sequence impedance for a rotating machine has been discussed in Chapter 12.

Let us represent positive, negative and zero sequence impedances respectively by Z_1, Z_2 and Z_0. We have already mentioned that for the symmetrical systems there is no mutual coupling between the sequence networks. The three-sequence systems can then be considered separately and phase currents and voltages determined by superposing their symmetrical components of current and voltage respectively.

Before we proceed further to use the symmetrical components technique for the analysis of unbalanced conditions in power systems, it is desirable to know the methods for measuring the sequence impedances.

Measurement of Sequence Impedances of Rotating Machines

Measurement of Positive Sequence Impedance: As already mentioned, the positive sequence impedance depends upon the working of the machine, *i.e.*, whether it is working under subtransient, transient or steady state condition. The impedance under steady state condition is known as the synchronous impedance and is measured by the well-known open circuit short circuit test. This impedance is defined as

$$\text{Synchronous impedance in p.u.} = \frac{\begin{array}{l}\text{Field current at rated armature current}\\ \text{on sustained symmetrical short circuit}\end{array}}{\begin{array}{l}\text{Field current at normal open circuit voltage on}\\ \text{the air gap line (\textit{i.e.}, the extended straight line}\\ \text{part of the magnetisation curve)}\end{array}}$$

Method of Test for Synchronous Impedance: The machine is run at synchronous speed in proper direction with the help of a prime mover (Fig. 13.2).

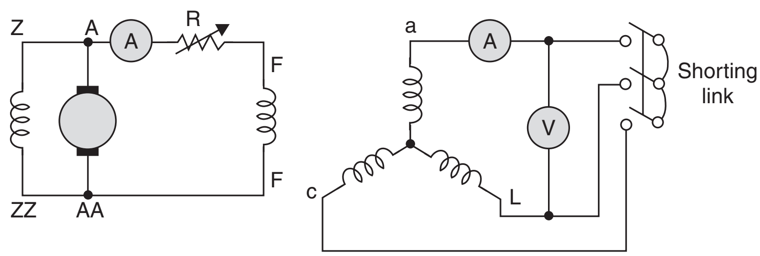


Fig. 13.2 Connection diagram for open circuit and short circuit test on an alternator.

The switch is kept in off position to perform open circuit test. The readings of voltmeter for various field currents are taken. Next the excitation is reduced to minimum by putting the total resistance in the field circuit and the switch is closed to perform short circuit test. Since short circuit test is under unsaturated condition of the machine it will be a linear characteristic passing through the origin and one single reading is enough. The two characteristics are plotted and according to the definition of synchronous impedance the value is calculated from the graph.

Method of Test for Subtransient Reactance: Apply voltage across any two terminals except the neutral with the rotor at rest and short circuited on itself through an ammeter (Fig. 13.3). The rotor is rotated by hand and it will be observed that for a fixed voltage applied, the current in the field varies with the position of the rotor. When the rotor is in the position of maximum induced field current (the direct axis position of rotor), one half the voltage required to circulate rated current is equal to the direct axis subtransient reactance X_d'' in per unit value. If the rotor is in the position of minimum induced field current the quadrature axis subtransient reactance X_q'' is obtained.

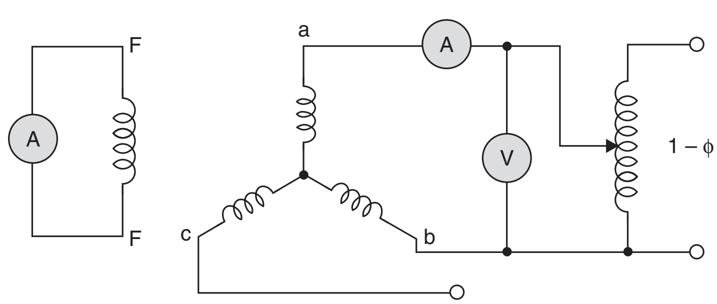


Fig. 13.3 Measurement of subtransient reactance of an alternator.

Measurement of Negative Sequence Reactance: The negative sequence reactance of a machine is the impedance offered to the flow of negative sequence current.

The machine is driven at rated speed and a reduced voltage is applied to circulate approximately the rated current. It is to be noted here that since negative sequence currents flow in this case, there is possibility of hunting which will result in oscillation of the pointer of the ammeter. The mean reading may be taken. The negative sequence impedance is given by

$$Z_2 = \frac{V}{\sqrt{3}I}$$

where V is the voltmeter and I the ammeter reading as shown in the diagram (Fig. 13.4).

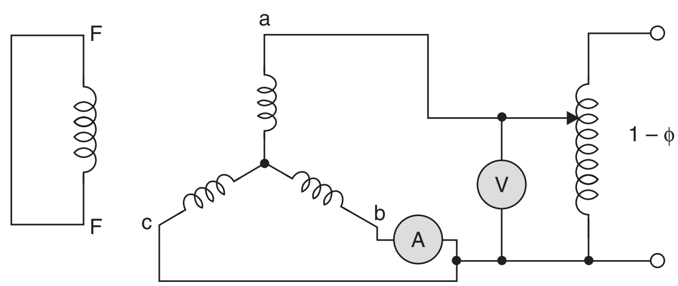


Fig. 13.4 Measurement of negative sequence impedance.

This can be proved mathematically as follows:

From the experiment, since it is similar to a line-to-line fault with alternator unloaded,

$$I_a = 0, I_b = I, I_c = -I$$

$$I_{a_1} = -I_{a_2} \quad \text{and} \quad V_{a_1} = V_{a_2} \quad \text{and} \quad V_{a_0} = 0,\ I_{a_0} = 0 \text{ (see section 13.7.1)}$$

From the measurement, voltage

$$V = V_a - V_b$$

i.e.

$$V = V_{a_1} + V_{a_2} - (\lambda^2 V_{a_1} + \lambda V_{a_2}) = 2V_{a_2} + V_{a_2} = 3V_{a_2}$$

and current in the ammeter

$$I = I_b = \lambda^2 I_{a_1} + \lambda I_{a_2} = (\lambda - \lambda^2) I_{a_2}$$

Now $(\lambda - \lambda^2) = -0.5 + j0.866 + 0.5 + j0.866 = j\sqrt{3} = |\sqrt{3}|\angle 90°$

$\therefore$ Current measured $= I = \sqrt{3}\, I_{a_2}$

Now
$$\frac{V}{\sqrt{3}I} = \frac{V_a - V_b}{\sqrt{3}I_b} = \frac{3V_{a_2}}{\sqrt{3}\cdot\sqrt{3}I_{a_2}} = \frac{V_{a_2}}{I_{a_2}} = Z_2$$

Measurement of Zero Sequence Impedance: Zero sequence impedance is the impedance offered by the machine to the flow of the zero sequence current. This impedance is quite variable and depends upon the distribution, *i.e.*, the pitch and the breadth factors. If the windings were infinitely distributed so that each phase produced a sinusoidal distribution of the m.m.f. then the superposition of the three phases with equal instantaneous currents cancel each other and produce zero field and consequently zero reactance except for slot and end-connection fluxes. The departure from this by introducing chording and breadth factors determines the zero sequence impedance. However, zero sequence impedance is much smaller than positive and negative sequence impedances. The machine must, of course, be star connected for otherwise the term zero sequence impedance has no significance as no zero sequence currents can flow.

The machine (Fig. 13.5) is at standstill and a reduced voltage is applied. The zero sequence impedance $Z_0 = V/3I$.

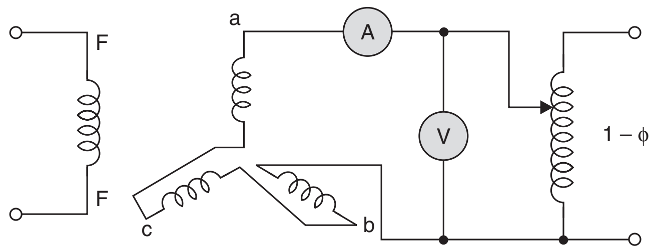


Fig. 13.5 Measurement of zero sequence impedance.

This connection ensures equal distribution of current in the three phases and for this reason is preferable to connecting the three phases in parallel. However, if the six terminals are not available the three phases are connected in parallel and experiment is conducted in the same fashion.

13.5 FAULT CALCULATIONS

Broadly speaking the faults can be classified as:

1. Shunt faults (short circuits).
2. Series faults (open conductor).

Shunt type of faults involve power conductor or conductors-to-ground or short circuit between conductors. When circuits are controlled by fuses or any device which does not open all three phases, one or two phases of the circuit may be opened while the other phases or phase is closed. These are called series type of faults. These faults may also occur with one or two broken conductors. Shunt faults are characterised by increase in current and fall in voltage and frequency whereas series faults are characterised by increase in voltage and frequency and fall in current in the faulted phases.

Shunt type of faults are classified as (*i*) Line-to-ground fault; (*ii*) Line-to-line fault; (*iii*) Double line-to-ground fault; and (*iv*) 3-phase fault. Of these, the first three are the unsymmetrical faults as the symmetry is disturbed in one or two phases. The method of symmetrical components will be utilized to analyse the unbalancing in the system. The 3-phase fault is a balanced fault which could also be analysed using symmetrical components.

The series faults are classified as: (*i*) one open conductor, and (*ii*) two open conductors. These faults also disturb the symmetry in one or two phases and are, therefore, unbalanced faults. The method of symmetrical components can be used for analysing such situations in the system.

Here we will discuss only the shunt type of faults.

Voltage of the Neutral

The potential of the neutral when it is grounded through some impedance or is isolated, will not be at ground potential under unbalanced conditions such as unsymmetrical faults. The potential of the neutral is given as $V_n = -I_n Z_n$, where Z_n is the neutral grounding impedance and I_n the neutral current. Here negative sign is used as the current flows from the ground to the neutral of the system and potential of the neutral is lower than the ground.

For a 3-phase system,

$$\begin{aligned} I_n &= I_a + I_b + I_c \\ &= (I_{a_1} + I_{a_2} + I_{a_0}) + (\lambda^2 I_{a_1} + \lambda I_{a_2} + I_{a_0}) + (\lambda I_{a_1} + \lambda^2 I_{a_2} + I_{a_0}) \\ &= I_{a_1}(1 + \lambda + \lambda^2) + I_{a_2}(1 + \lambda + \lambda^2) + 3 I_{a_0} \\ &= 3 I_{a_0} \qquad (13.16) \end{aligned}$$

$$\therefore \quad V_n = -3 I_{a_0} Z_n \qquad (13.17)$$

Since the positive sequence and negative sequence components of currents through the neutral are absent, the drops due to these currents are also zero. Also for a balanced set of currents or voltages the neutral is at ground potential; therefore, for positive and negative sequence networks, neutral of the system will be taken as the reference.

Reference of Voltages

The phase voltages at any point in a grounded system and their zero sequence components of voltage will be referred to the ground at that point. The positive and negative sequence components of voltage are referred to neutral. For the positive and negative sequence systems, therefore, the expressions voltage to neutral and voltage to ground may be used interchangeably but for the zero sequence system it is important to distinguish between the two terms.

The analysis here will apply to a symmetrical 3-phase system with dissymmetry only at one point *i.e.*, faults at simultaneously more than one point will not be considered. In a 3-phase system, the unknown quantities are the 3-phase voltage V_a, V_b and V_c and the 3-phase currents I_a, I_b and I_c *i.e.*, there are six unknowns. To determine these quantities, six linearly independent equations are required. In any given problem, certain conditions are required about the unknown quantities and these are the boundary conditions which can be expressed in the form of equation, *e.g.*, if conductor a is faulted to ground at some point, the voltage of this conductor at the faulted point is zero, *i.e.*, $V_a = 0$. It has already been seen that the 3-phase voltages and currents can be expressed in terms of their corresponding three symmetrical components. Therefore, instead of 3-phase voltages and currents being unknown one can say that six symmetrical components V_{a_0}, V_{a_1}, V_{a_2}, I_{a_0}, I_{a_1} and I_{a_2} are unknown. In a 3-phase system, three equations (boundary conditions) can be written in terms of the three unknown phase currents and voltages at the point of dissymmetry. Three more equations are needed for a solution of the six unknowns. The advantage in using the six unknown components instead of the six unknown phase quantities is that the impedances met by the sequence currents can be determined either by calculation or test. This is not usually the case with phase impedances. However, if the phase impedances can also be readily obtained, there may be no advantage in introducing components; in fact, the use of phase quantities may give a simpler solution. The three sequence equations using the sequence generated voltages and the sequence impedances are derived as follows.

13.6 SEQUENCE NETWORK EQUATIONS

These equations will be derived for an unloaded alternator with neutral solidly grounded, assuming that the system is balanced, *i.e.*, the generated voltages are of equal magnitude and displaced by 120°. Consider the diagram (Fig. 13.6).

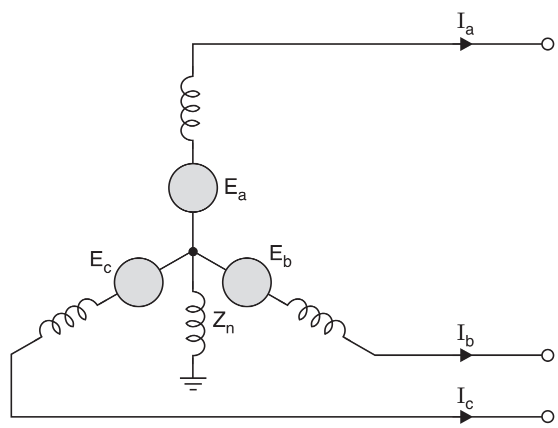


Fig. 13.6 A balanced 3-phase system.

Since the sequence impedances per phase are same for all three phases and we are considering initially a balanced system the analysis will be done on single phase basis. The positive sequence component of voltage at the fault point is the positive sequence generated voltage minus the drop due to positive sequence current in positive sequence impedance (as positive sequence current does not produce drop in negative or zero sequence impedances)

$$V_{a_1} = E_a - I_{a_1} Z_1$$

Similarly, the negative sequence component of voltage at the fault point is the generated negative sequence voltage minus the drop due to negative sequence current in negative sequence impedance (as negative sequence current does not produce drop in positive or zero sequence impedances)

$$V_{a_2} = E_{a_2} - I_{a_2} Z_2$$

Since the negative sequence voltage generated is zero, therefore,

$$E_{a_2} = 0$$

or

$$V_{a_2} = - I_{a_2} Z_2$$

Similarly, for zero sequence voltages

$$E_{a_0} = 0$$

$$V_{a_0} = V_n - I_{a_0} Z_{g_0} = -3 I_{a_0} Z_n - I_{a_0} Z_{g_0} = - I_{a_0} (Z_{g_0} + 3Z_n)$$

where Z_{g_0} is the zero sequence impedance of the generator and Z_n is the neutral impedance.

The three sequence network equations are, therefore,

$$V_{a_1} = E_a - I_{a_1} Z_1 \qquad (13.18)$$

$$V_{a_2} = -I_{a_1}Z_2 \quad (13.19)$$

$$V_{a_0} = -I_{a_0}Z_0 \quad (13.20)$$

where $Z_0 = Z_{g_0} + 3Z_n$ and the corresponding sequence networks for the unloaded alternator are shown in Fig. 13.7.

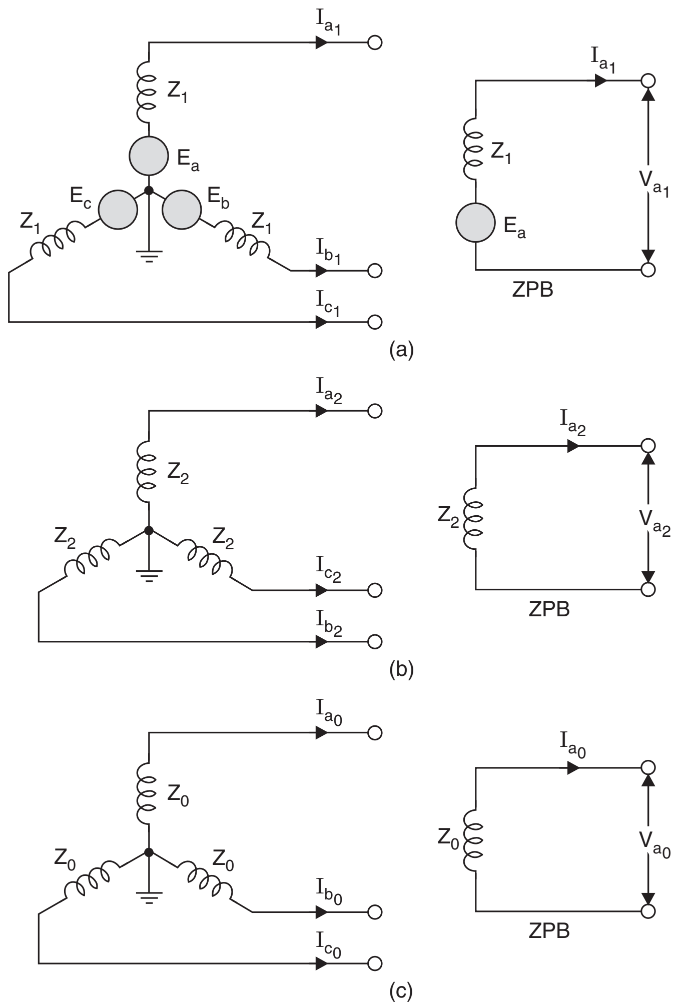


Fig. 13.7 Sequence networks: (*a*) Positive sequence network; (*b*) Negative sequence network; and (*c*) Zero sequence network.

Simultaneous solution of the three sequence equations and the three boundary conditions equations in which the phase quantities have been replaced by their symmetrical components of currents and voltages, will give the six unknown symmetrical components of currents and voltages. Once the symmetrical components of currents and voltages are known the phase currents and voltages can be obtained by using the relation (13.7) through (13.9) respectively. The sequence network equation in matrix notation will be

$$\begin{bmatrix} V_{a_0} \\ V_{a_1} \\ V_{a_2} \end{bmatrix} = \begin{bmatrix} 0 \\ E_a \\ 0 \end{bmatrix} - \begin{bmatrix} Z_0 & 0 & 0 \\ 0 & Z_1 & 0 \\ 0 & 0 & Z_2 \end{bmatrix} \begin{bmatrix} I_{a_0} \\ I_{a_1} \\ I_{a_2} \end{bmatrix} \qquad (13.20a)$$

Now we are ready with mathematical tools to analyse various types of shunt faults. For all type of faults the sequence network equations will be as given by equations (13.18)–(13.20) whereas the three equations describing the boundary conditions will be different for different types of faults. The analysis will be done by both the algebraic manipulations and the matrix manipulations for the sake of completeness. We will analyse first of all a system where faults take place on an unloaded alternator with neutral solidly grounded and it is assumed that the faults are also solid so that no impedance is introduced between the fault points. Later on the analysis will be made with (*i*) neutral grounded through some impedance Z_n, and (*ii*) fault having some impedance Z_f.

13.7 SINGLE LINE-TO-GROUND FAULT

The system to be analysed is shown in Fig. 13.8. Let the fault take place on phase *a*. The boundary conditions are

$$V_a = 0 \qquad (13.21)$$

$$I_b = 0 \qquad (13.22)$$

$$I_c = 0 \qquad (13.23)$$

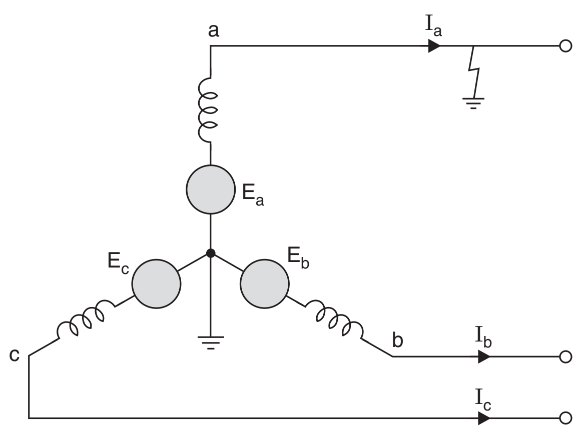


Fig. 13.8 A solidly grounded, unloaded alternator: L-G fault on phase a.

and the sequence network equations are

$$V_{a_0} = -I_{a_0} Z_0 \qquad (13.18)$$

$$V_{a_1} = E_a - I_{a_1} Z_1 \qquad (13.19)$$

$$V_{a_2} = -I_{a_2} Z_2 \qquad (13.20)$$

The solution of these six equations will give six unknowns V_{a_0}, V_{a_1}, V_{a_2} and I_{a_0}, I_{a_1} and I_{a_2}.

From equation (13.13),

$$I_{a_1} = \tfrac{1}{3}(I_a + \lambda I_b + \lambda^2 I_c)$$

$$I_{a_2} = \tfrac{1}{3}(I_a + \lambda^2 I_b + \lambda I_c)$$

$$I_{a_0} = \tfrac{1}{3}(I_a + I_b + I_c)$$

Substituting the values of I_b and I_c from equations (13.22–13.23),

$$I_{a_1} = I_{a_2} = I_{a_0} = I_a/3 \tag{13.24}$$

Equation (13.21) can be written in terms of symmetrical components

$$V_a = 0 = V_{a_1} + V_{a_2} + V_{a_0} \tag{13.25}$$

Now substituting the values of V_{a_0}, V_{a_1} and V_{a_2} from the sequence network equation,

$$E_a - I_{a_1} Z_1 - I_{a_2} Z_2 - I_{a_0} Z_0 = 0 \tag{13.26}$$

Since $\quad I_{a_1} = I_{a_2} = I_{a_0}$

Equation (13.26) becomes

$$E_a - I_{a_1} Z_1 - I_{a_1} Z_2 - I_{a_1} Z_0 = 0$$

or

$$I_{a_1} = \frac{E_a}{Z_1 + Z_2 + Z_0} \tag{13.27}$$

From equation (13.27) it is clear that to simulate a *L-G* fault all the three sequence networks are required and since the currents are all equal in magnitude and phase angle, therefore, the three sequence networks must be connected in series. The voltage across each sequence network corresponds to the same sequence component of V_a. The interconnection of the sequence network is shown in Fig. 13.9.

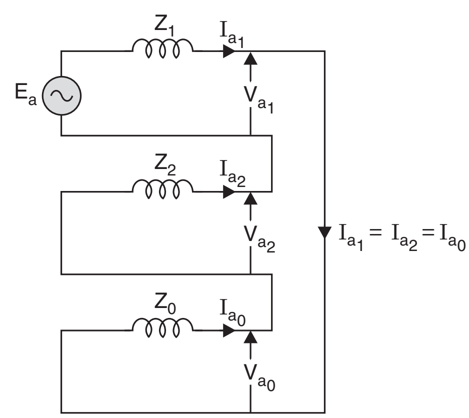


Fig. 13.9 Interconnection of sequence networks for L-G fault.

So far we have calculated $I_{a_1} = I_{a_2} = I_{a_0}$. To calculate the remaining three unknowns $V_{a_0}, V_{a_1}, V_{a_2}$, use is made of the sequence network equations.

The analysis will now be made using matrix manipulations.

From equation (13.13)

$$\begin{bmatrix} I_{a_0} \\ I_{a_1} \\ I_{a_2} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \lambda & \lambda^2 \\ 1 & \lambda & \lambda^2 \end{bmatrix}\begin{bmatrix} I_a \\ 0 \\ 0 \end{bmatrix}$$

Substituting for $I_b = I_c = 0$,

$$\begin{bmatrix} I_{a_0} \\ I_{a_1} \\ I_{a_2} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \lambda & \lambda^2 \\ 1 & \lambda & \lambda^2 \end{bmatrix}\begin{bmatrix} I_a \\ 0 \\ 0 \end{bmatrix}$$

From this equation $\quad I_{a_0} = I_{a_1} = I_{a_2} = I_a/3$

Substituting equation (13.24) into equation (13.20(*a*)),

$$\begin{bmatrix} V_{a_0} \\ V_{a_1} \\ V_{a_2} \end{bmatrix} = \begin{bmatrix} 0 \\ E_a \\ 0 \end{bmatrix} - \begin{bmatrix} Z_0 & 0 & 0 \\ 0 & Z_1 & 0 \\ 0 & 0 & Z_2 \end{bmatrix}\begin{bmatrix} I_{a_1} \\ I_{a_1} \\ I_{a_1} \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ E_a \\ 0 \end{bmatrix} - \begin{bmatrix} I_{a_1} Z_0 \\ I_{a_1} Z_1 \\ I_{a_1} Z_2 \end{bmatrix}$$

$$\begin{bmatrix} V_{a_0} \\ V_{a_1} \\ V_{a_2} \end{bmatrix} = \begin{bmatrix} -I_{a_1} Z_0 \\ E_a - I_{a_1} Z_1 \\ -I_{a_1} Z_2 \end{bmatrix}$$

$$\therefore \qquad V_{a_0} + V_{a_1} + V_{a_2} = 0 = -I_{a_1} Z_0 + E_a - I_{a_1} Z_1 - I_{a_1} Z_2$$

$$\therefore \qquad I_{a_1} = \frac{E_a}{Z_1 + Z_2 + Z_0}$$

Now in case of line-to-ground fault the neutral current

$$I_n = I_a = I_{a_1} + I_{a_2} + I_{a_0}$$

and for the same case, $\quad I_{a_1} = I_{a_2} = I_{a_0}$

$$\therefore \qquad I_n = 3 I_{a_0}$$

In case the neutral is not grounded the zero sequence impedance Z_0 becomes infinite and, therefore, from equation (13.27),

$$I_{a_1} = \frac{E_a}{Z_1 + Z_2 + \infty} = 0$$

The same result can be envisaged by looking at the system when the neutral is isolated; there is no return path for the current and, therefore, $I_{a_1} = I_{a_2} = I_{a_0} = 0$. This means that for this system the fault current $I_a = 0$.

Example 13.3: A 25 MVA, 13.2 kV alternator with solidly grounded neutral has a subtransient reactance of 0.25 p.u. The negative and zero sequence reactances are 0.35 and 0.1 p.u. respectively. A single line to ground fault occurs at the terminals of an unloaded alternator; determine the fault current and the line-to-line voltages. Neglect resistance.

Solution: Normally the positive sequence impedance is greater than the negative sequence but since the given positive sequence impedance corresponds to the subtransient state, it may be less than the negative sequence impedance. The sequence network for a line-to-ground fault current is shown in Fig. E.13.3.

Let the line-to-neutral voltage at the fault point before the fault be 1.0 + j0.0 p.u. For a line-to-ground fault the fault impedance is

$$j0.25 + j0.35 + j0.1 = j0.7$$

$$\therefore \quad I_{a_1} = \frac{E_a}{Z_1 + Z_2 + Z_0} = \frac{1 + j0.0}{j0.7} = -j1.428$$

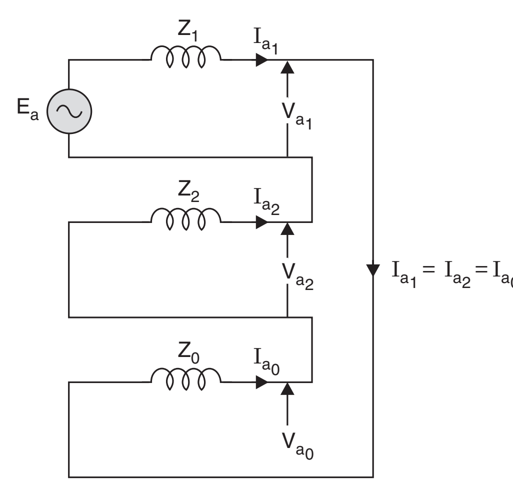


Fig. E.13.3 Interconnection of sequence network.

For a *L-G* fault

$$I_{a_1} = I_{a_2} = I_{a_0} = -j1.428$$

$\therefore$ The p.u. fault current $\quad I_a = I_{a_1} + I_{a_2} + I_{a_0} = 3I_{a_1} = -j4.285$

Let the base quantities be 25 MVA, 13.2 kV, and hence

$$\text{the base current} = \frac{25 \times 1000}{\sqrt{3} \times 13.2} = 1093 \text{ amps}$$

$\therefore$ The fault current in amperes = 1093 × 4.285 = 4685 amps

To find out the voltages, we first find out the sequence components of voltages.

$$V_{a_1} = E_a - I_{a_1} Z_1$$
$$= 1 + j0.0 - (-j1.428)(j0.25)$$
$$= 1 - 0.357 = 0.643$$
$$V_{a_2} = -I_{a_2} Z_2 = -(-j1.428)(j0.35)$$
$$= -0.4998$$

Similarly, $\quad V_{a_0} = -I_{a_0} Z_0 = -(-j1.428)(j0.1) = 0.1428$

As a numeric check $V_a = 0$. Substituting the values of V_{a_1}, V_{a_2} and V_{a_0},

$$0.643 - 0.4998 - 0.1428 \simeq 0$$
$$V_b = V_{b_1} + V_{b_2} + V_{b_0} \text{ and } V_c = V_{c_1} + V_{c_2} + V_{c_0}$$

Now $\quad V_{b_1} = \lambda^2 V_{a_1} = (-0.5 - j0.866)(0.643)$

$$= -0.3215 - j0.5568$$
$$V_{b_2} = \lambda V_{a_2} = (-0.5 + j0.866)(-0.50)$$
$$= (0.25 - j0.433)$$
$$V_{b_0} = V_{a_0} = V_{c_0} = -0.1428$$
$$V_{c_1} = \lambda V_{a_1} = (-0.5 + j0.866)(0.643)$$
$$= -0.3215 + j0.5568$$
$$V_{c_2} = \lambda^2 V_{a_2} = (-0.5 - j0.866)(-0.5)$$
$$= 0.25 + j0.433$$

$\therefore \quad V_b = -0.3215 - j0.5568 + 0.25 - j0.433 - 0.1428$

$$= -0.2143 - j0.9898$$

and $\quad V_c = -0.3215 + j0.5568 + 0.25 + j0.433 - 0.1428$

$$= -0.2143 + j0.9898$$

Now the line-to-line voltage

$$V_{ab} = V_a - V_b. \text{ Since } V_a = 0,$$
$$V_{ab} = -V_b = 0.2143 + j0.9898$$
$$V_{ac} = -V_c = 0.2143 - j0.9898$$

and $\quad V_{bc} = V_b - V_c = -j2 \times 0.9898$

$$= -j1.9796$$

Now $$V_{ab} = 0.2143 + j0.9898 = \sqrt{(0.4592 + 9.797) \times 10^{-1}}$$

$$= \sqrt{10.346 \times 10^{-1}} = \sqrt{1.0346} = 1.0127 \text{ p.u.}$$

The line-to-line voltage will be

$$V_{ab} = 1.0127 \times \frac{13.2}{\sqrt{3}} = 7.717 \text{ kV}$$

$$V_{ac} = 7.717 \text{ kV}$$

and $$V_{bc} = 1.9796 \times \frac{13.2}{\sqrt{3}} = 15.08 \text{ kV.}$$

Line-to-line Fault

As shown in Fig. 13.10, the line-to-line fault takes place on phases b and c. The boundary conditions are

$$I_a = 0 \tag{13.28}$$

$$I_b + I_c = 0 \tag{13.29}$$

$$V_b = V_c \tag{13.30}$$

and the sequence network equations are given by equations (13.18)–(13.20). The solution of these six equations will give six unknowns.

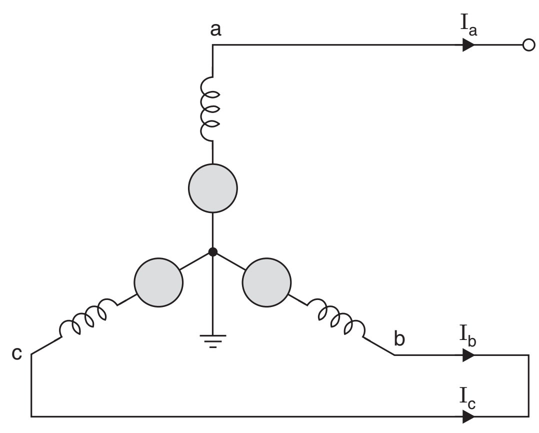


Fig. 13.10 L-L fault on an unloaded and neutral grounded alternator.

Using the relations $$I_{a_1} = \tfrac{1}{3}(I_a + \lambda I_b + \lambda^2 I_c)$$

$$I_{a_2} = \tfrac{1}{3}(I_a + \lambda^2 I_b + \lambda I_c)$$

$$I_{a_0} = \tfrac{1}{3}(I_a + I_b + I_c)$$

and substituting for I_a, I_b and I_c

$$I_{a_1} = \tfrac{1}{3}(0 + \lambda I_b - \lambda^2 I_b)$$

$$= \tfrac{1}{3}(\lambda - \lambda^2) I_b$$

$$I_{a_2} = \tfrac{1}{3}(0 + \lambda^2 I_b - \lambda I_b)$$

$$= \frac{I_b}{3}\,(\lambda^2 - \lambda)$$

and $$I_{a_0} = \tfrac{1}{3}(0 + 0) = 0$$

which means for a line-to-line fault the zero-sequence component of current is absent and positive-sequence component of current is equal in magnitude but opposite in phase to negative sequence component of current, *i.e.*

$$I_{a_1} = - I_{a_2} \qquad \text{...(13.31)}$$

To simulate *L-L* fault condition zero sequence network is not required and the positive and negative-sequence networks are to be connected in opposition as $I_{a_1} = - I_{a_2}$.

Now from equations (13.8) and (13.9)

$$V_b = V_{a_0} + \lambda^2 V_{a_1} + \lambda V_{a_2}$$
$$V_c = V_{a_0} + \lambda V_{a_1} + \lambda^2 V_{a_1}$$

Substituting these relations in equation (13.30),

$$V_{a_0} + \lambda^2 V_{a_1} + \lambda V_{a_2} = V_{a_0} + \lambda V_{a_1} + \lambda^2 V_{a_2}$$

or $$(\lambda^2 - \lambda) V_{a_1} = (\lambda^2 - \lambda) V_{a_2}$$

$\therefore$ $$V_{a_1} = V_{a_2} \qquad \text{...(13.32)}$$

That is, positive-sequence component of voltage equals the negative-sequence component of voltage. This also means that the two sequence networks are connected in opposition. Now making use of the sequence network equation and the equation (13.32),

$$V_{a_1} = V_{a_2}$$
$$E_a - I_{a_1} Z_1 = - I_{a_2} Z_2 = I_{a_1} Z_2$$

or $$I_{a_1} = \frac{E_a}{Z_1 + Z_2}$$

The interconnection of the sequence network for simulation of *L-L* fault is shown in Fig. 13.11.

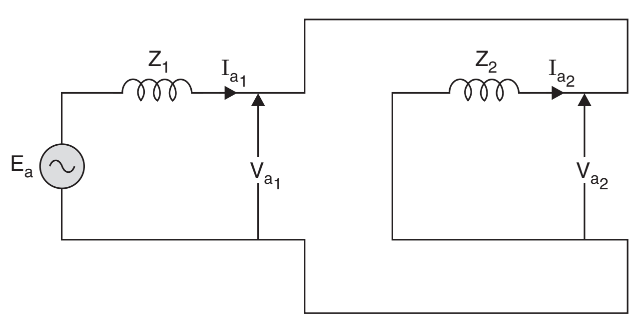


Fig. 13.11 Interconnection of sequence networks for L-L fault.

So far we have calculated I_{a_1}, I_{a_2} and I_{a_0}, we can calculate the three symmetrical components of voltages V_{a_1}, V_{a_2} and V_{a_0} and then using the relations (13.7)–(13.9), the phase currents and voltages can be obtained. It is to be noted here that since $I_{a_0} = 0$, $\therefore$ $V_{a_0} = 0$.

The *L-L* fault can be analysed using matrix manipulation as follows:

Using the relation (14.13) and substituting for I_a, I_b and I_c,

$$\begin{bmatrix} I_{a_0} \\ I_{a_1} \\ I_{a_2} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \lambda & \lambda^2 \\ 1 & \lambda^2 & \lambda \end{bmatrix}\begin{bmatrix} 0 \\ I_b \\ -I_b \end{bmatrix}$$

$$I_{a_0} = 0,\ I_{a_1} = (\lambda - \lambda^2)I_b \quad \text{and} \quad I_{a_2} = (\lambda^2 - \lambda)I_b$$

$$\therefore \qquad I_{a_1} = -I_{a_2}$$

Again using the relation (13.20*a*) and substituting for V_a, V_b and V_c,

$$\begin{bmatrix} V_{a_0} \\ V_{a_1} \\ V_{a_2} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \lambda & \lambda^2 \\ 1 & \lambda^2 & \lambda \end{bmatrix}\begin{bmatrix} V_2 \\ V_b \\ V_b \end{bmatrix}$$

$$V_{a_0} = \tfrac{1}{3}(V_a + V_b + V_c) = 0$$

$$V_{a_1} = \tfrac{1}{3}(V_a + \lambda V_b + \lambda^2 V_b)$$

$$V_{a_2} = \tfrac{1}{3}(V_a + \lambda^2 V_b + \lambda V_b)$$

$$\therefore \qquad V_{a_1} = V_{a_2}$$

The sequence network equations are

$$\begin{bmatrix} 0 \\ V_{a_1} \\ V_{a_1} \end{bmatrix} = \begin{bmatrix} 0 \\ E_a \\ 0 \end{bmatrix} - \begin{bmatrix} Z_0 & 0 & 0 \\ 0 & Z_1 & 0 \\ 0 & 0 & Z_2 \end{bmatrix}\begin{bmatrix} 0 \\ I_{a_1} \\ -I_{a_1} \end{bmatrix}$$

$$\therefore \qquad V_{a_1} = E_a - I_{a_1}Z_1 = +\, I_{a_1}Z_2$$

$$\therefore \qquad I_{a_1} = \frac{E_a}{Z_1 + Z_2}.$$

The interconnection of the sequence network for simulating *L-L* fault satisfies all the relations derived. We have derived mathematically that zero sequence current will be absent in this case, which can be envisaged physically from the network also. We see that in the system there is only one ground *i.e.*, the grounded neutral of the system and since the fault does not involve ground the zero sequence currents which are single phase currents do not flow *i.e.*, $I_{a_0} = 0$.

Example 13.4: Determine the fault current and the line-to-line voltage at the fault when a line-to-line fault occurs at the terminals of the alternator described in Example 13.3.

Solution: The sequence network for *L-L* fault is shown in Fig. E.13.4. Since the zero sequence network is absent, assuming $(1 + j0.0)$ prefault per unit voltage,

$$I_{a_1} = \frac{E_a}{Z_1 + Z_2} = \frac{1 + j0.0}{j0.25 + j0.35}$$

$$= \frac{1 + j0.0}{j0.6} = -j1.667$$

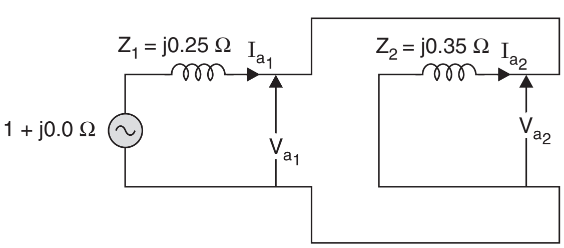


Fig. E.13.4 Sequence network.

Now for a *L-L* fault

$$I_{a_1} = -I_{a_2} = -j1.667$$

∴ $$I_{a_2} = j1.667$$

and $$I_{a_0} = 0$$

To find out the fault current, $I_b = -I_c$, we use the following relations:

$$I_b = I_{b_1} + I_{b_2} + I_{b_0} = I_{b_1} + I_{b_2} = \lambda^2 I_{a_1} + \lambda I_{a_2}$$
$$= (-0.5 - j0.866)(-j1.667) + (-0.5 + j0.866)(j1.667)$$
$$= j0.833 - 1.4436 - j0.833 - 1.4436$$
$$= -2.8872 \text{ p.u.}$$

Now base current is 1093 amperes.

∴ Fault current = 1093 × 2.8872 = 3155.71 amperes

To find out line-to-line voltage we find out the sequence components of voltages

$$V_{a_1} = E_a - I_{a_1} Z_1 = 1 + j0.0 - (-j1.667)(j0.25)$$
$$= 1 - 0.4167 = 0.5833$$

Similarly,

$$V_{a_2} = -I_{a_2} Z_2 = (-j1.667)(j0.35) = 0.5834$$

i.e., $$V_{a_1} = V_{a_2} = 0.5833 \text{ p.u.}$$

$$V_a = V_{a_1} + V_{a_2} + V_{a_0} = V_{a_1} + V_{a_2} = 2 \times 0.5833 = 1.1666 \text{ p.u.}$$
$$V_b = \lambda^2 V_{a_1} + \lambda V_{a_2}$$
$$= (-0.5 - j0.866)\,(0.5833) + (-0.5 + j0.866)\,(0.5833)$$
$$= -0.5833$$

and $$V_b = V_c = -0.5833$$

Line voltage

$$V_{ab} = V_a - V_b = 1.1666 - (-0.5833) = 1.7499$$
$$V_{ac} = V_a - V_c = 1.7499$$

and $$V_{bc} = V_b - V_c = 0.0$$

The line-to-line voltage

$$V_{ab} = 1.7499 \times \frac{13.2}{\sqrt{3}} = 13.33 \text{ kV}$$
$$V_{ac} = 13.33 \text{ kV}$$

and $$V_{bc} = 0.0 \text{ kV.}$$ **Ans.**

Double Line to Ground Fault

Double line to ground fault takes place on phases *b* and *c* (Fig. 13.12). The boundary conditions are

$$I_a = 0 \quad (13.33)$$
$$V_b = 0 \quad (13.34)$$
$$V_c = 0 \quad (13.35)$$

and the sequence network equations are given by (13.18)–(13.20).

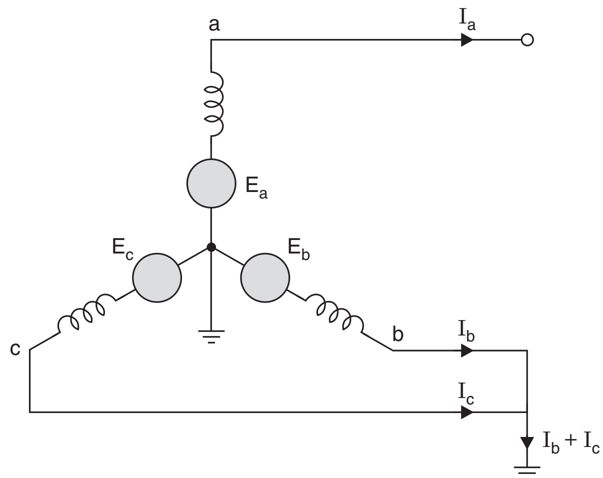


Fig. 13.12 A solidly grounded, unloaded alternator, L-L-G fault.

The solution of these six equations will give the six unknown symmetrical components.

Using the equations (13.10)–(13.12) and substituting for V_a, V_b and V_c from (13.34) and (13.35).

$$V_{a_0} = \tfrac{1}{3}(V_a + V_b + V_c)$$
$$= V_a/3$$
$$V_{a_1} = \tfrac{1}{3}(V_a + \lambda V_b + \lambda^2 V_c)$$
$$= V_a/3$$
$$V_{a_2} = \tfrac{1}{3}(V_a + \lambda^2 V_b + \lambda V_c)$$
$$= V_a/3$$

i.e.,
$$V_{a_0} = V_{a_1} = V_{a_2} \tag{13.36}$$

Using this relation of voltages and substituting in the sequence network equations

$$V_{a_0} = V_{a_1}$$
$$-I_{a_0}Z_0 = E_a - V_{a_1}Z_1$$

∴
$$I_{a_0} = -\frac{E_a - I_{a_1}Z_1}{Z_0} \tag{13.37}$$

Similarly
$$V_{a_2} = V_{a_1}$$
$$-I_{a_2}Z_2 = E_a - I_{a_1}Z_1$$

∴
$$I_{a_2} = -\frac{E_a - I_{a_1}Z_1}{Z_2} \tag{13.38}$$

Now from equation (13.33),

$$I_a = I_{a_1} + I_{a_2} + I_{a_0} = 0$$

Substituting values of I_{a_2} and I_{a_0} from equations (13.38) and (13.37),

$$I_{a_1} - \frac{E_a - I_{a_1}Z_1}{Z_2} - \frac{E_a - I_{a_1}Z_2}{Z_0} = 0$$

Rearranging the terms gives

$$I_{a_1} = \frac{E_a}{Z_1 + \dfrac{Z_0 Z_2}{Z_0 + Z_2}} \quad \text{...(13.39)}$$

From equation (13.39) it is clear that all the three sequence networks are required to simulate *L-L-G* fault and also that the negative and zero sequence networks are connected in parallel. The sequence network interconnection is shown in Fig. 13.13.

From equation (13.39) it is clear that the zero and negative sequence networks are first connected in parallel and then in opposition with the positive sequence network. The same has been shown in Fig. 13.13.

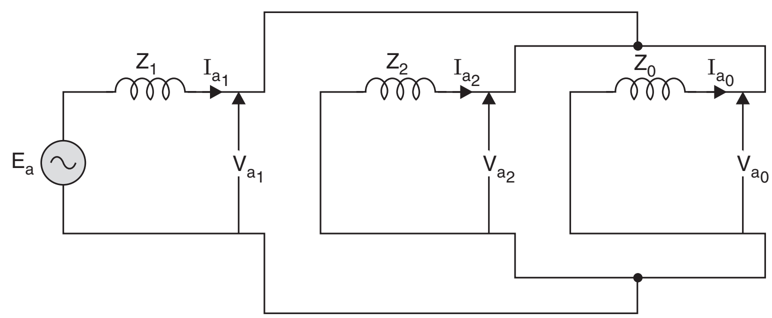


Fig. 13.13 Interconnection of sequence networks for L-L-G fault.

The analysis is made using matrix manipulation.

$$\begin{bmatrix} V_{a_0} \\ V_{a_1} \\ V_{a_2} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \lambda & \lambda^2 \\ 1 & \lambda^2 & \lambda \end{bmatrix}\begin{bmatrix} V_a \\ 0 \\ 0 \end{bmatrix}$$

$\therefore$ $$V_{a_0} = V_{a_1} = V_{a_2} = V_a/3$$

Using these relations in the sequence network equations,

$$\begin{bmatrix} V_{a_1} \\ V_{a_1} \\ V_{a_1} \end{bmatrix} = \begin{bmatrix} 0 \\ E_a \\ 0 \end{bmatrix} - \begin{bmatrix} Z_0 & 0 & 0 \\ 0 & Z_1 & 0 \\ 0 & 0 & Z_2 \end{bmatrix}\begin{bmatrix} I_{a_0} \\ I_{a_1} \\ I_{a_2} \end{bmatrix}$$

These equations are to be solved for I_{a_0}, I_{a_1} and I_{a_1}.

Rearranging the terms,

$$\begin{bmatrix} Z_0 & 0 & 0 \\ 0 & Z_1 & 0 \\ 0 & 0 & Z_2 \end{bmatrix}\begin{bmatrix} I_{a_0} \\ I_{a_1} \\ I_{a_2} \end{bmatrix} = \begin{bmatrix} -V_{a_1} \\ E_1 - V_{a_1} \\ -V_{a_1} \end{bmatrix}$$

or $$AX = B$$

where X is the current vector.

So to find X, pre-multiply this equation by A^{-1}. Therefore,

$$X = A^{-1}B.$$

Now
$$\begin{bmatrix} Z_0 & 0 & 0 \\ 0 & Z_1 & 0 \\ 0 & 0 & Z_2 \end{bmatrix}^{-1} = \begin{bmatrix} 1/Z_0 & 0 & 0 \\ 0 & 1/Z_1 & 0 \\ 0 & 0 & 1/Z_2 \end{bmatrix}$$

as it is a diagonal matrix.

Therefore,
$$\begin{bmatrix} I_{a_0} \\ I_{a_1} \\ I_{a_2} \end{bmatrix} = \begin{bmatrix} 1/Z_0 & 0 & 0 \\ 0 & 1/Z_1 & 0 \\ 0 & 0 & 1/Z_2 \end{bmatrix} \begin{bmatrix} -V_{a_1} \\ E_a - V_{a_1} \\ -V_{a_1} \end{bmatrix}$$

or
$$I_{a_0} = -\frac{V_{a_1}}{Z_0} = -\frac{E_a - I_{a_1} Z_1}{Z_0}$$

$$I_{a_2} = -\frac{V_{a_1}}{Z_2} = -\frac{E_a - I_{a_1} Z_1}{Z_2}$$

Use the relation $I_{a_1} + I_{a_2} + I_{a_0} = 0$ and substitute the values of I_{a_0} and I_{a_2} as in equations (13.37) and (13.38) and rearrange the terms. The following is obtained:

$$I_{a_1} = \frac{E_a}{Z_1 + \dfrac{Z_0\, Z_2}{Z_0 + Z_2}}$$

The neutral current

$$\begin{aligned} I_n &= I_b + I_c \\ &= \lambda^2 I_{a_1} + \lambda I_{a_2} + I_{a_0} + \lambda I_{a_1} + \lambda^2 I_{a_2} + I_{a_0} \\ &= (\lambda^2 + \lambda) I_{a_1} + (\lambda + \lambda^2) I_{a_2} + 2 I_{a_0} \\ &= - I_{a_1} - I_{a_2} + 2 I_{a_0} \\ &= I_{a_0} + 2 I_{a_0} = 3 I_{a_0} \end{aligned} \tag{13.40}$$

Example 13.5: Determine the fault current and the line-to-line voltages at the fault when a double line-to-ground fault occurs at the terminals of the alternator described in Example 13.4.

Solution: Assuming $(1 + j0.0)$ p.u. as prefault voltage,

$$I_{a_1} = \frac{E_a}{Z_1 + \dfrac{Z_0\, Z_2}{Z_0 + Z_2}} = \frac{1 + j0.0}{j0.25 + \dfrac{j0.1 \times j0.35}{j0.45}} = \frac{1 + j0.0}{j0.25 + j0.0778}$$

$$= \frac{1 + j0.0}{j0.3278} = -j3.0506 \text{ p.u.}$$

Now for L-L-G, $\quad V_{a_1} = V_{a_2} = V_{a_0}$

Also $\quad V_{a_1} = E_a - I_{a_1} Z_1$

To find out I_{a_2} and I_{a_0}, we should first find V_{a_1} and since $V_{a_1} = V_{a_2} = -I_{a_2} Z_2$, I_{a_2} can be obtained.

Similarly, $\quad V_{a_1} = V_{a_0} = -I_{a_0} Z_0$, I_{a_0} can be obtained.

$$V_{a_1} = 1 + j0.0 - (-j3.0506)(j0.25)$$
$$= 1 - 0.7626 = 0.2374$$

$\therefore$ $$V_{a_2} = V_{a_0} = 0.2374$$

and $$I_{a_2} = -\frac{V_{a_2}}{Z_2} = -\frac{0.2374}{j0.35} = j\frac{0.2374}{0.35} = j0.678$$

Similarly, $$I_{a_0} = -\frac{V_{a_0}}{Z_0} = -\frac{0.2374}{j0.1} = j2.374$$

$$I_{a_2} + I_{a_0} = j0.678 + j2.374 = j3.05 = -I_{a_1}$$

Now fault current $= I_b + I_c = 3I_{a_0} = 3 \times j2.374 = j7.122$ p.u.

Since base current is 1093 amperes, the fault current will be

$$1093 \times 7.122 = 7784.3 \text{ amperes}$$

$$V_a = V_{a_1} + V_{a_2} + V_{a_0} = 3V_{a_1} = 3 \times 0.2374 = 0.7122$$

and $V_b = V_c = 0$

The line-to-line fault voltage,

$$V_{ab} = V_a = 0.7122 \times \frac{13.2}{\sqrt{3}} = 5.42 \text{ kV}$$

$$V_{ac} = V_a = 0.7122 \times \frac{13.2}{\sqrt{3}} = 5.42 \text{ kV}$$

$$V_{bc} = 0.0 \text{ kV}$$

3-phase Fault

As shown in Fig. 13.14, the boundary conditions are

$$I_a + I_b + I_c = 0 \quad (13.41)$$

$$V_a = V_b = V_c \quad (13.42)$$

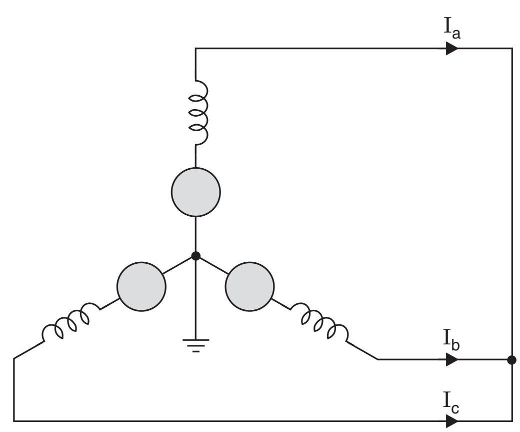


Fig. 13.14 A 3-phase neutral grounded and unloaded alternator 3-phase shorted.

Since $|I_a| = |I_b| = |I_c|$ and if I_a is taken as reference

$$I_b = \lambda^2 I_a \quad \text{and} \quad I_c = \lambda I_a$$

Using the relation

$$I_{a_1} = \tfrac{1}{3}(I_a + \lambda I_b + \lambda^2 I_c)$$

and substituting the values of I_b and I_c,

$$I_{a_1} = \tfrac{1}{3}(I_a + \lambda^3 I_a + \lambda^3 \mathrm{I}_a)$$

$$= I_a \tag{13.43}$$

$$I_{a_2} = \tfrac{1}{3}(I_a + \lambda^2 I_b + \lambda I_c)$$

Substituting for I_b and I_c in terms of I_a,

$$I_{a_2} = \tfrac{1}{3}(I_a + \lambda^4 I_a + \lambda^2 I_a)$$

$$= \tfrac{1}{3}(I_a + \lambda I_a + \lambda^2 I_a)$$

$$= \frac{I_a}{3}(1 + \lambda + \lambda^2)$$

$$= 0 \tag{13.44}$$

Similarly,

$$I_{a_0} = \tfrac{1}{3}(I_a + I_b + I_c)$$

$$= 0 \tag{13.45}$$

which means that for a 3-phase fault zero-as well as negative-sequence components of current are absent and the positive-sequence component of current is equal to the phase current.

Now using the voltage boundary relation,

$$V_{a_1} = \tfrac{1}{3}(V_a + \lambda V_b + \lambda^2 V_c) = \tfrac{1}{3}(V_a + \lambda V_a + \lambda^2 V_a)$$

$$= \frac{V_a}{3}(1 + \lambda + \lambda^2) = 0 \tag{13.46}$$

$$V_{a_2} = \tfrac{1}{3}(V_a + \lambda^2 V_b + \lambda V_c)$$

$$= 0 \tag{13.47}$$

$$V_{a_0} = 0 \tag{13.48}$$

Since

$$V_{a_1} = 0 = E_a - I_{a_1} Z_1,$$

$\therefore$

$$I_{a_1} = \frac{E_a}{Z_1} \tag{13.49}$$

The sequence network is shown in Fig. 13.15.

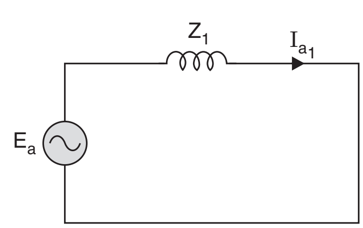


Fig. 13.15 Interconnection of sequence network-3-phase fault.

From the analysis of the various faults, the following observations are made:

1. Positive sequence currents are present in all types of faults.

2. Negative sequence currents are present in all unsymmetrical faults.

3. Zero sequence currents are present when the neutral of the system is grounded and the fault also involves the ground, and magnitude of the neutral current is equal to $3I_{a_0}$.

Since only the positive sequence voltages are generated in the synchronous machine, the question is frequently raised as to the origin of negative and zero sequence voltages that appear throughout the network. It is seen from the analysis that any unbalanced condition gives rise to positive sequence currents and other sequence currents. The negative-and zero-sequence currents produce corresponding drops in their respective networks. These voltages are in general a maximum at the fault point and decrease as the neutral bus is approached.

So far we have studied the various faults on an unloaded alternator with the neutral solidly grounded and the fault is assumed to be solid, *i.e.*, with no fault impedance. Now we will analyse all these faults with neutral impedance Z_n and fault impedance Z_f. Analysis will be made using algebraic manipulations only. Matrix method will not be repeated, the reader can always try the analysis based on the treatment done earlier in this chapter.

13.8 LINE-TO-GROUND FAULT WITH Z_f

The fault impedance is Z_f and the neutral impedance Z_n (Fig. 13.16).

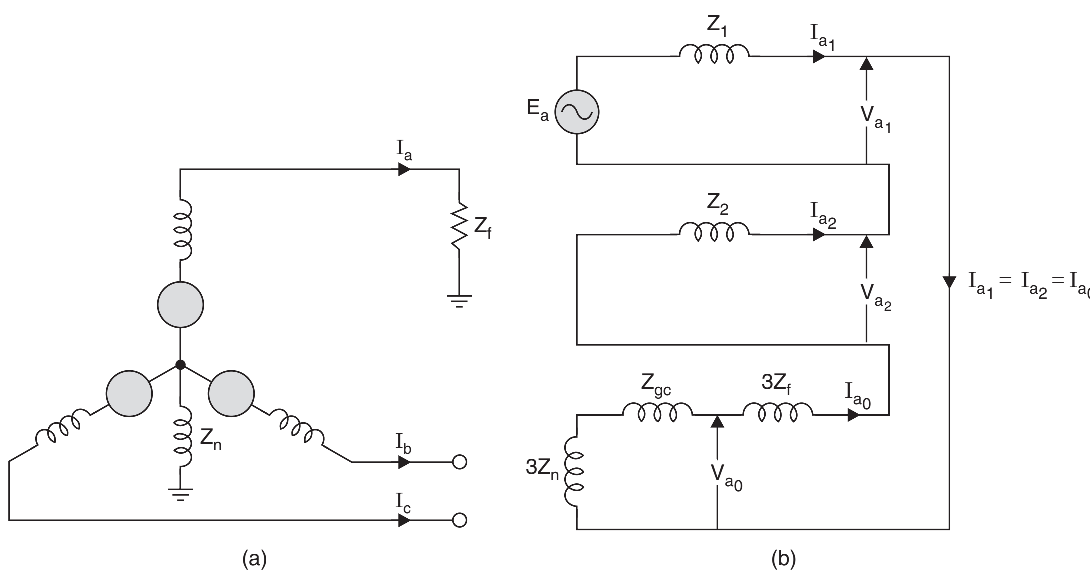


Fig. 13.16 (*a*) A 3-phase unloaded alternator with neutral grounded through impedance Z_n and fault impedance Z_f, L-G fault; (*b*) Interconnection of sequence network for L-G fault.

The boundary conditions are

$$V_a = I_a Z_f$$
$$I_b = 0, I_c = 0$$
$$V_{a_0} = - I_{a_0} (Z_{g_0} + 3Z_n)$$
$$V_{a_1} = E_a - I_{a_1} Z_1, V_{a_2} = - I_{a_2} Z_2$$

The solution of these equations gives the unknown quantities.

From equation (13.13) and the boundary condition above,

$$I_{a_1} = I_{a_2} = I_{a_0} = I_a/3$$
$$V_{a_1} + V_{a_2} + V_{a_0} = V_a = 3 I_{a_1} (Z_f)$$
$$E_a - I_{a_1} Z_1 - I_{a_1} Z_2 - I_{a_1} (Z_0 + 3Z_n) = 3 I_{a_1} (Z_f)$$

$\therefore$ $$E_a = I_{a_1} [Z_1 + Z_2 + \{(Z_0 + 3Z_n) + 3Z_f\}]$$

$\therefore$ $$I_{a_1} = \frac{E_a}{Z_1 + Z_2 + (Z_0 + 3Z_n) + 3Z_f} \quad (13.50)$$

Since I_{a_1}, I_{a_2} and I_{a_0} are known, V_{a_1}, V_{a_2} and V_{a_0} can be calculated from the sequence network equations. The sequence network interconnection is shown in Fig. 13.16(*b*).

Line-to-Line Fault with Z_f

The boundary conditions, as shown in Fig. 13.17(*a*), are

$$I_a = 0 \quad (13.28)$$
$$I_b + I_c = 0 \quad (13.29)$$
$$V_b = V_c + I_b Z_f \quad (13.51)$$

and the sequence network equations are

$$V_{a_1} = E_a - I_{a_1} Z_1$$
$$V_{a_2} = - I_{a_2} Z_2$$
$$V_{a_0} = - I_{a_0} Z_0$$

By using equation (13.13), we know that $I_{a_1} = - I_{a_2}$ and $I_{a_0} = 0$.

Using equations (13.8)–(13.9) in equation (13.51),

$$V_b = V_c + I_b Z_f$$
$$V_{a_0} + \lambda^2 V_{a_1} + \lambda V_{a_2} = V_{a_0} + \lambda V_{a_1} + \lambda^2 V_{a_2} + (\lambda^2 I_{a_1} + \lambda I_{a_2}) Z_f$$

or $$\lambda^2 V_{a_1} - \lambda V_{a_1} = (\lambda^2 - \lambda) V_{a_2} + (\lambda^2 I_{a_1} - \lambda I_{a_1}) Z_f$$

or $$V_{a_1} = V_{a_2} + I_{a_1} Z_f \quad (13.52)$$

Now substituting for V_{a_1} and V_{a_2} from the sequence network equations,

$$E_a - I_{a_1} Z_1 = - I_{a_2} Z_2 + I_{a_1} Z_f$$
$$E_a - I_{a_1} Z_1 = I_{a_1} (Z_2 + Z_f)$$

or $$I_{a_1} = \frac{E_a}{Z_1 + (Z_2 + Z_f)} \quad (13.53)$$

The interconnection of the sequence network is shown in Fig. 13.17(*b*).

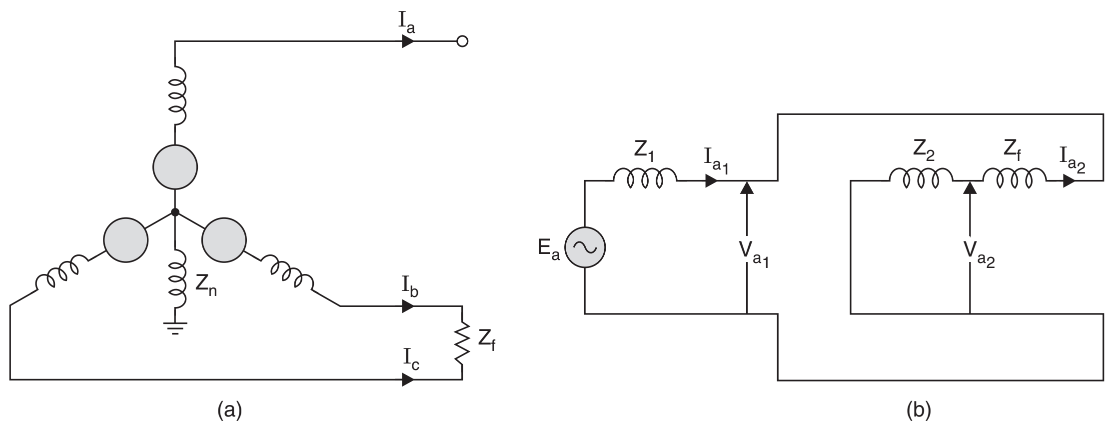


Fig. 13.17 (*a*) L-L fault; (*b*) Interconnection of sequence network, fault impedance Z_f, L-L fault.

Double Line-to-Ground Fault

Fault impedance is Z_f and neutral impedance z_n. The boundary conditions, as shown in Fig. 13.18(*a*), are

$$I_b = 0$$

$$V_b = V_c = (I_b + I_c)Z_f \tag{13.54}$$

and the sequence network equations are

$$V_{a_1} = E_a - I_{a_1} Z_1$$

$$V_{a_2} = - I_{a_1} Z_2$$

$$V_{a_0} = - I_{a_0} (Z_0 + 3Z_n)$$

We know that $(I_b + I_c) = 3 I_{a_0}$

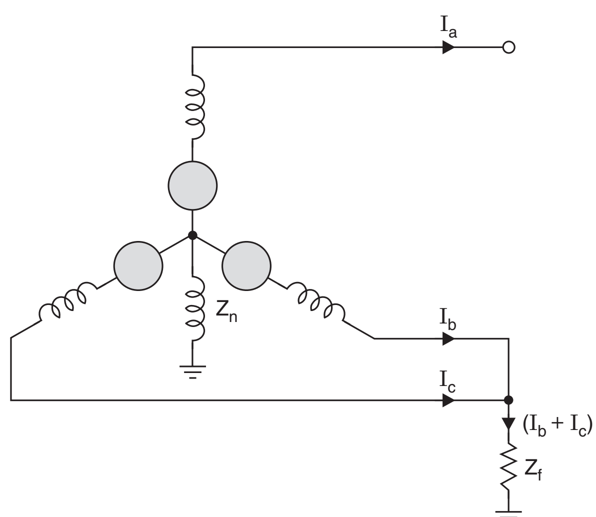


Fig. 13.18 (*a*) L-L-G fault. Fault impedance Z_f and neutral impedance Z_n.

∴ Equation (13.54) becomes

$$V_b = V_c = 3I_{a_0}Z_f$$

∴ $$\lambda^2 V_{a_1} + \lambda V_{a_2} + V_{a_0} = \lambda V_{a_1} + \lambda^2 V_{a_2} + V_{a_0}$$

or $$V_{a_1} = V_{a_2}$$

Using this relation in equation

$$V_b = 3I_{a_0} Z_f$$

$$\lambda^2 V_{a_1} + \lambda V_{a_1} + V_{a_0} = 3I_{a_0}Z_f$$

or $$-V_{a_1} + V_{a_0} = 3I_{a_0}Z_f$$

or $$V_{a_1} = V_{a_0} - 3I_{a_0}Z_f$$

Substituting for V_{a_1} and V_{a_0} from the sequence equation and expressing I_{a_0} in terms of I_{a_1}, we get

$$E_a - I_{a_1}Z_1 = -I_{a_0}(Z_0 + 3Z_n) - 3I_{a_0}Z_f$$

or $$I_{a_0} = -\frac{E_a - I_{a_1}Z_1}{Z_0 + 3Z_n + 3Z_f}$$

Similarly making use of the relation $V_{a_1} = V_{a_2}$, we express I_{a_2} in terms of I_{a_1}.

$$E_a - I_{a_1}Z_1 = -I_{a_2}Z_2$$

or $$I_{a_2} = -\frac{E_a - I_{a_1}Z_1}{Z_2}$$

Substituting the values of I_{a_2} and I_{a_0} in the equation

$$I_a = I_{a_1} + I_{a_2} + I_{a_0} = 0$$

$$I_{a_1} - \frac{E_a - I_{a_1}Z_1}{Z_2} - \frac{E_a - I_{a_1}Z_1}{Z_0 + 3Z_n + 3Z_f} = 0$$

or $$I_{a_1} = \frac{E_a}{Z_1 + \dfrac{Z_2(Z_0 + 3Z_n + 3Z_f)}{Z_2 + Z_0 + 3Z_n + 3Z_f}} \qquad (13.55)$$

The interconnection of the sequence network is shown in Fig. 13.18(*b*).

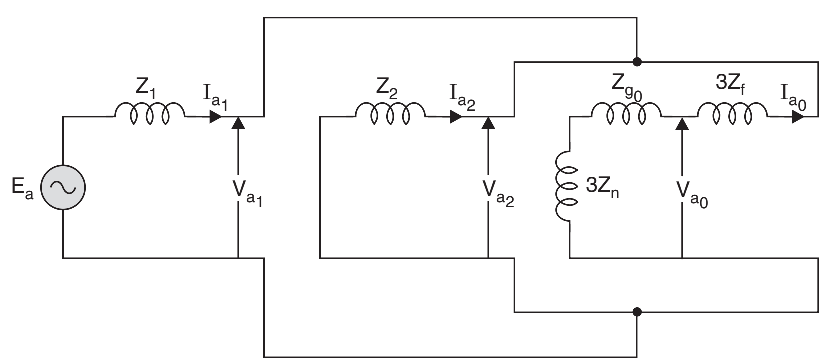


Fig. 13.18 (*b*) Interconnection of sequence networks for Fig. 13.18(*a*).

Before we proceed further to study the faults on an actual system where the alternator may be connected to a transmission line through a transformer or any other interconnected system, we will like to study the sequence network representation of various components like a generator, transformer, a synchronous motor etc.

13.9 SEQUENCE NETWORKS

The positive sequence network is in all respects identical with the usual networks considered. Each synchronous machine must be considered as a source of e.m.f. which may vary in magnitude and phase position depending upon the distribution of power and reactive volt amperes just prior to the occurrence of the fault. The positive sequence voltage at the point of fault will drop, the amount being dependent upon the type of faults; for 3-phase faults it will be zero; for double line-to-ground fault, line-to-line fault and single line-to-ground fault, it will be higher in the order stated.

The negative sequence network is in general quite similar to the positive sequence network except for the fact that since no negative sequence voltages are generated, the source of e.m.f. is absent.

The zero sequence network likewise will be free of internal voltages, the flow of current resulting from the voltage at the point of fault. The impedances to zero sequence current are very frequently different from the positive or negative sequence currents. The transformer and generator impedances will depend upon the type of connections whether star or delta connected; if star, whether grounded or not.

Equivalent circuit for the zero sequence network depends upon the impedances met by the zero sequence currents flowing through the three phases and their sum, $3I_{a_0}$, flowing through the neutral impedance and returning through the ground or a neutral conductor. If there is no complete path for zero sequence currents in a circuit, the zero sequence impedance is infinite. Thus a Y-connected circuit with ungrounded neutral has infinite impedance to zero sequence currents (Fig. 13.19(*a*)).

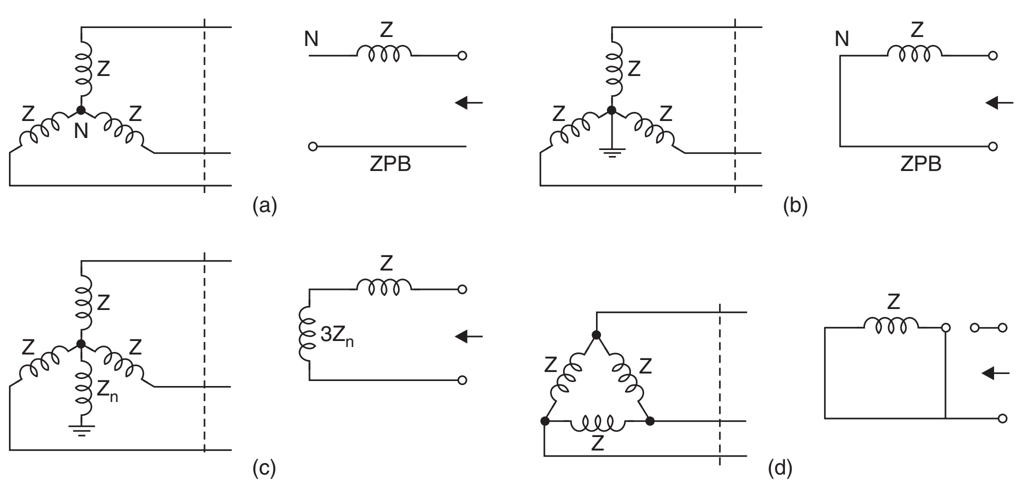


Fig. 13.19 Zero sequence networks for a 3-phase load.

In case the star point is solidly grounded *i.e.*, zero impedance between the neutral and the ground, a zero impedance is connected between the neutral point and the zero potential bus (Fig. 13.19(*b*)).

In case the neutral is grounded through some impedance Z_n, an impedance of $3Z_n$ should be connected between the neutral point and the zero potential bus (Fig. 13.21(*c*)).

A current of $3I_{a_0}$ produces a drop of $3I_{a_0}Z_n$ and to show in the equivalent zero sequence network the same drop where current of I_{a_0} flows, the impedance should be $3Z_n$.

A delta-connected circuit provides no path for zero sequence currents flowing in the line. The zero sequence currents being single phase, circulate within the winding. Hence viewed from its terminals its zero sequence impedance is infinite (Fig. 13.21(*d*)).

The zero sequence equivalent circuits of 3-phase transformers require special attention because of possibility of various combinations. The general circuit for any combination is given in Fig. 13.20 (*a*).

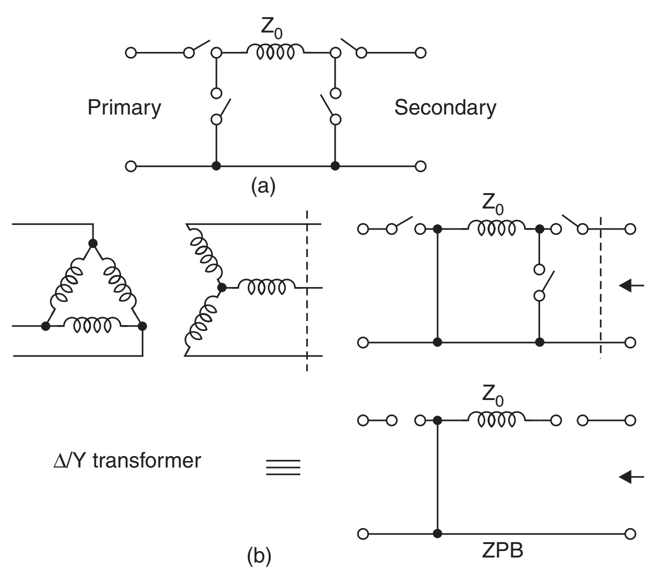


Fig. 13.20 (*a*) Switch arrangements for a transformer
(*b*) Equivalent of Δ/Y.

Z is the zero sequence impedance of the windings of the transformer. These are two series and two shunt switches. See the location of the switches. One series and one shunt switch are for both the sides separately. The series switch of a particular side is closed if it is star grounded and the shunt switch is closed if that side is delta connected, otherwise they are left open.

Say the transformer is *Δ/Y* connected with star ungrounded (Fig. 13.20(*b*)). Since the primary is delta connected, the shunt switch of primary side is closed and series is left open. The secondary is star ungrounded; therefore, the series switch is left open and shunt switch is also left open.

The zero sequence equivalent circuits for a few more combinations using this rule are drawn in Fig. 13.21.

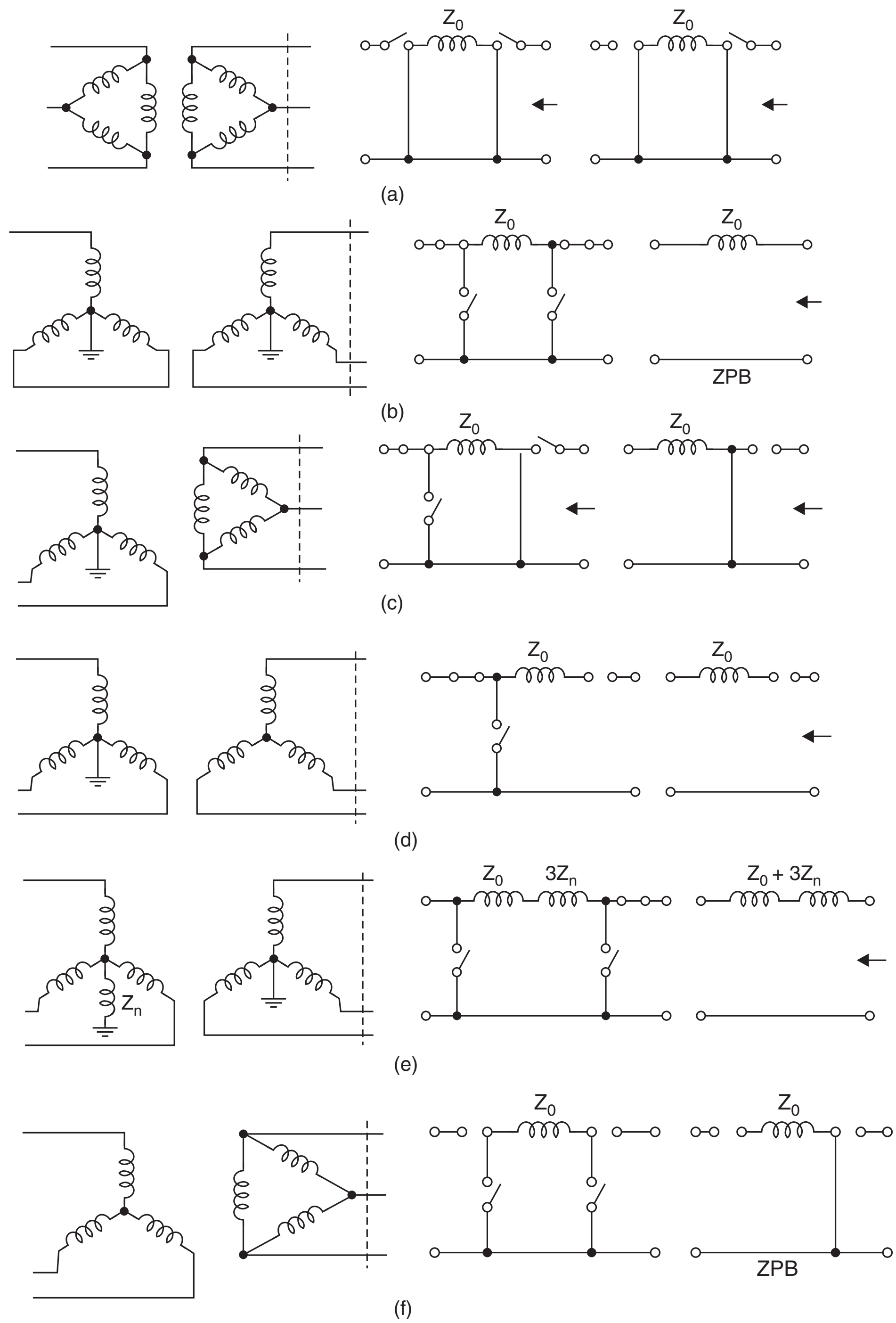


Fig. 13.21 Zero sequence equivalent circuits of transformers.

The reader after having some practice with the switch diagram will be able to draw the equivalent circuit very easily. Now we are ready to analyse the faults on power system.

13.10 FAULTS ON POWER SYSTEMS

The faults are analysed easily by making use of Thevenin's theorem. As the readers know that this theorem can be used for determining the changes that take place in currents and voltages of a linear network when an additional impedance is added between two nodes of the network. The theorem states that:

The changes that take place in the network voltages and currents due to the addition of an impedance (a short circuit) between two network nodes are identical with those voltages and currents that would be caused by an e.m.f. placed in series with the impedance and having a magnitude and polarity equal to the pre-fault voltage that existed between the nodes in question and the impedance as seen between the nodes with all active voltage sources short circuited.

To determine the current and voltage distribution in the system, the distribution in each of the sequence networks must first be determined. The Thevenin's equivalents of positive, negative and zero sequence networks are identical to those of a network of single generator.

Consider the system in Fig. 13.22 for illustration of the application of Thevenin's theorem for determining the equivalent positive, negative and zero sequence networks.

Thevenin's equivalent of positive sequence networks is obtained from the positive sequence network. The Thevenin's equivalent voltage source is the prefault voltage at the fault point and the equivalent impedance $Z_{1\,eq}$ is the impedance as seen between the fault point and the zero potential bus shorting the voltage sources. It is to be noted here that positive sequence impedance of the alternator or the synchronous machine depends upon the state of the machine *i.e.*, whether it is sub-transient, transient or steady state.

Similarly, the Thevenin's equivalent negative and zero sequence networks are obtained from the negative and zero sequence networks respectively. Since the system is balanced, no negative or zero sequence currents are flowing before the fault occurs. The prefault negative and zero sequence voltages at the fault point are zero. Therefore, no e.m.fs. appear in the equivalent circuits. The impedances $Z_{2\,eq}$ and $Z_{0\,eq}$ are measured between the fault point and the reference bus in their respective networks.

In the positive network, the currents throughout the system due to the fault can be added to the load currents before the fault to give the total positive sequence current during the fault. The net fault current is the fault current considering the system under no load condition plus the load current super-imposed over the fault currents.

(a)

Fig. 13.22 (*a*) Single line diagram of a balanced 3-phase system.

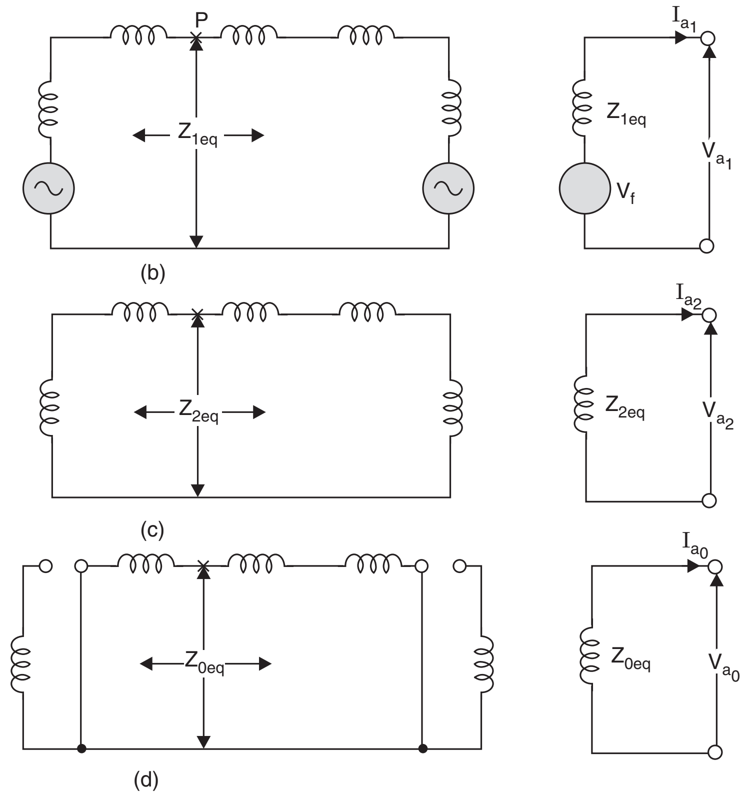


Fig. 13.22 (*b*), (*c*) and (*d*) Thevenin's equivalent of positive, negative and zero sequence networks.

13.11 PHASE SHIFT Δ-Y TRANSFORMERS

The two possible ways of connecting Δ-*Y* transformers are shown in Figs. 13.23 (*a*) and (*b*).

The small letters used refer to the star side and capital letters to the delta side of the transformer. The winding *e′e* on star side corresponds to the *E′E* on the delta side. The primed letters indicate the beginning of the winding and unprimed the finish of the winding. Figs. 13.23 (*c*) and (*d*) give the voltage vector diagram for positive sequence of the connections in (*a*) and (*b*) respectively, neglecting the voltage drop in the transformer. Say vector diagram (*c*) is drawn such that V_{a_1} and V_{CB1} are in phase and the other vectors follow. Similarly, in (*d*), V_{a_1} and V_{BC_1} are in phase. If each voltage is expressed in per unit with its own voltage as the base voltage, V_{BC_1}, V_{a_1} and V_{A_1} in Fig. (*c*) are equal in magnitude, and therefore,

$$V_{A_1} = jV_{BC_1} = jV_{a_1} \tag{13.56}$$

whereas in Fig. (*d*)

$$V_{A_1} = -jV_{BC_1} = -jV_{a_1} \tag{13.57}$$

From the above, it is clear that the line to neutral voltage V_{A_1} on the delta side leads the line to neutral voltage on star side in Fig. (*a*) by 90° whereas in Fig. (*b*) it lags by 90°.

The connection diagram in Figs. (*a*) and (*b*) and their corresponding vector diagrams for positive sequence voltage in Figs. (*c*) and (*d*) relate to the usual transformer connection diagrams and hence if the connection diagram is given, the phase relation between V_{A_1} and V_{a_1} can be determined by inspection. Referring to Figs. (*e*) and (*f*) which are the negative sequence voltage vector diagrams of Figs. (*a*) and (*b*) respectively, we have

For Fig. (*e*),

$$V_{A_2} = -jV_{CB_2} = -jV_{a_2} \tag{13.58}$$

and for Fig. (*f*),

$$V_{A_2} = jV_{BC_2} = jV_{a_2} \tag{13.59}$$

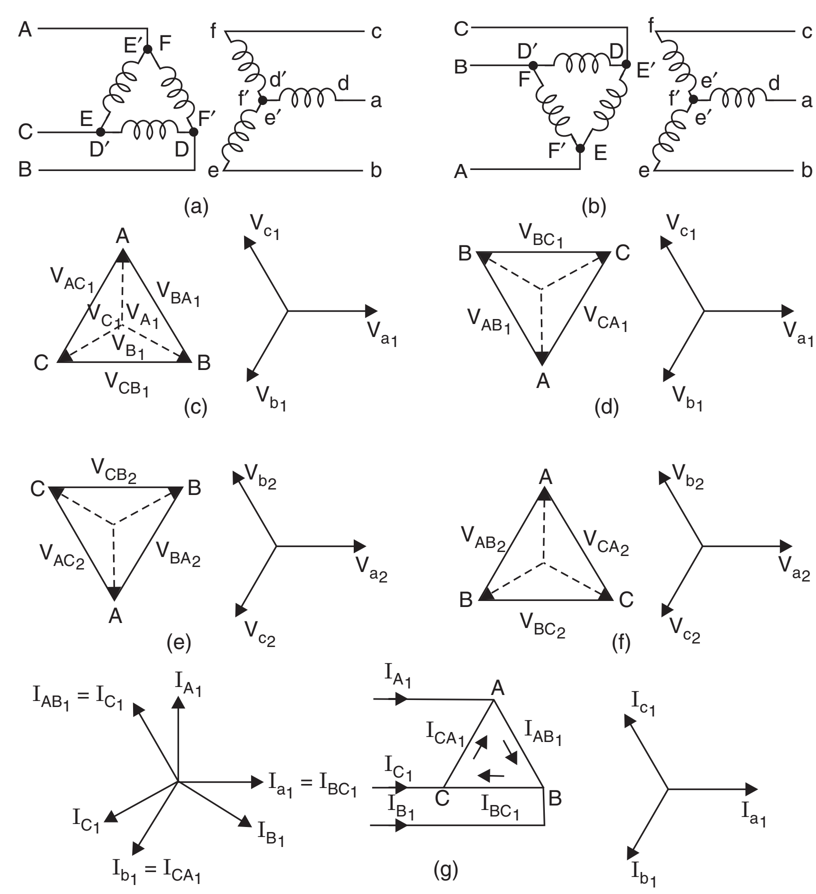


Fig. 13.23 Phase shift in Δ-Y transformer.

It is clear that the phase shift in the negative sequence voltages is in the direction opposite to the shift in phase of the positive sequence voltage for the same connection diagram.

Since the kVA rating of the transformer on the two sides is the same, if we neglect the exciting current, resistance and the voltage drop, it is essential that the shift in phase of positive and negative sequence line currents in passing through a Δ-*Y* or *Y*-Δ transformer banks with transformer exciting currents neglected must correspond to the shift in phase of line-to-neutral voltages with the drop neglected.

Referring of Fig. (*g*) which corresponds to the positive sequence current vector diagram of Fig. (*a*), let the currents leave the neutral of the star side and enter the delta side of the transformer. This means in star, the current goes from *e′* to *e* whereas in delta it goes from *E* to *E′*, *i.e.,* from *B* to *C* as indicated by the arrow. Arrows on the delta side are used to indicate direction of current flow but do not indicate the direction of phase relation with respect to star currents. Let I_{a_1} be the reference vector and with exciting current neglected I_{BC_1} is in phase with I_{a_1}. Again expressing the line currents in per unit with its own-current as the base current

$$I_{A_1} = -jI_{CB_1} = jI_{BC_1} = jI_{a_1} \tag{13.60}$$

Similarly for negative sequence current,

$$I_{A_2} = -jI_{a_2} \tag{13.61}$$

In fact these current relations can be derived in a different way also. We know that the total input to the transformer as a unit is zero assuming a lossless transformer, *i.e.,* $V_1I_1 + V_2I_2 = 0$. That is

$$V_{A_1} I_{A_1} + V_{a_1} I_{a_1} = 0 \tag{13.62}$$

Now we have from equation (13.56),

$$V_{A_1} = jV_{a_1}$$

Substituting this relation in equation (13.62),

or $$jV_{a_1} I_{A_1} + V_{a_1} I_{a_1} = 0$$

or $$jI_{A_1} = -I_{a_1}$$

and $$I_{A_1} = jI_{a_1} \tag{13.60}$$

or $$V_{A_2} I_{A_2} + V_{a_2} I_{a_2} = 0$$

or $$-jV_{a_2} I_{A_2} + V_{a_2} I_{a_2} = 0$$

or $$-jI_{A_2} = -I_{a_2}$$

$$I_{A_2} = -jI_{a_2} \tag{13.61}$$

Similarly for the other connections where $V_{A_1} = -jV_{a_1}$ and $V_{A_2} = jV_{a_2}$ the current relations can be derived.

It is, therefore, seen that the positive sequence line-to-neutral voltages and line currents are shifted 90° in phase in the same direction in passing through a *Y*-Δ or Δ-*Y* transformer whereas the corresponding negative sequence quantities are shifted 90° in the direction opposite to the positive sequence shift.

In case it is desired to know only the magnitude of voltage and currents in a system during faults, we need not consider the phase shift of 90°. If both magnitude and phase relations are required then we must consider the 90° phase shift. To solve the short circuit problems in which the connection of the Δ-*Y* transformer is not given, any one of the two connections can be assumed. The only difference in the final results will be the sign of the voltages and currents. The sign in one case is plus and in the other it will be minus, the magnitudes will remain same.

Example 13.6: A 30 MVA, 13.8 kV, 3-phase alternator has a subtransient reactance of 15% and negative and zero sequence reactances of 15% and 5% respectively. The alternator supplies two motors over a transmission line having transformers at both ends as shown on the one-line diagram. The motors have rated inputs of 20 MVA and 10 MVA both 12.5 kV with

20% subtransient reactance and negative and zero sequence reactances are 20% and 5% respectively. Current limiting reactors of 2.0 ohms each are in the neutral of the alternator and the larger motor. The 3-phase transformers are both rated 35 MVA, 13.2 Δ-115*Y* kV with leakage reactance of 10%. Series reactance of the line is 80 ohms. The zero sequence reactance of the line is 200 ohms. Determine the fault current when (*i*) *L-G* (*ii*) *L-L*, and (*iii*) *L-L-G* fault takes place at point *P*. Assume V_f = 120 kV.

Solution: The three sequence networks will be as shown in Fig. E.13.6. Assume base of 30 MVA and base voltage of 13.8 kV in generator circuit.

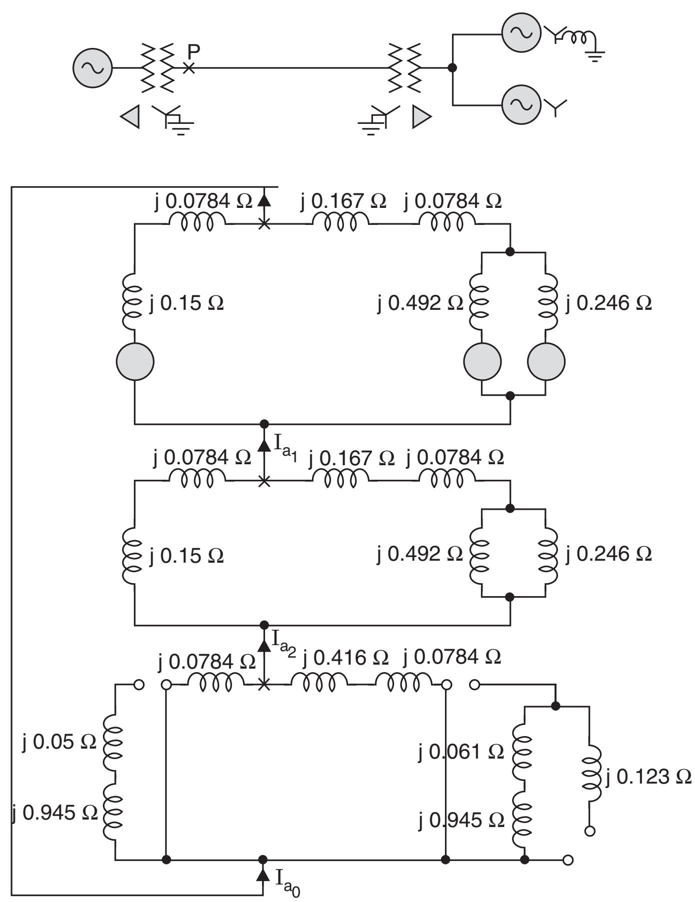


Fig. E.13.6

Positive Sequence Network

The base voltage on the line side of the transformer = $13.8 \times \dfrac{115}{13.2} = 120$ kV

∴ The base voltage on the motor side of the transformer = $120 \times \dfrac{13.2}{115} = 13.8$ kV

The per cent reactance of transformer = $10 \times \left(\frac{13.2}{13.8}\right)^2 \times \frac{30}{35}$ = 7.8423%

The per cent reactance of motor = $20 \times \left(\frac{12.5}{13.8}\right)^2 \times \frac{30}{20}$ = 24.6%

The per cent reactance of line = $80 \times \frac{30}{120^2} \times 100 = \frac{2400}{144}$ = 16.7%

Negative Sequence Network: The network is exactly identical to positive sequence network except for the sources.

Zero Sequence Network

The neutral reactance = $2 \times 3 \times \frac{30}{(13.8)^2} \times 100 = \frac{180 \times 100}{(13.8)^2}$ = 94.5%

The zero sequence reactance of line = $200 \times \frac{30}{(120)^2} \times 100 = \frac{6000}{144}$ = 41.6%

Once the three sequence networks are ready we analyse different fault conditions as follows:

L-G Fault: The three sequence networks are connected in series, positive sequence impedance between *P* and *ZPB* is (when sources are short circuited) j0.146. Similarly,

Negative sequence impedance = j0.146

Zero sequence impedance = 0.06767

Total impedance = j0.3596

$$\therefore \qquad I_{a_1} = \frac{1 + j0.0}{j0.35967} = -j2.78 \text{ p.u.} = I_{a_2} = I_{a_0}$$

Fault current = $3I_{a_1} = -j8.34$

Base current = $\frac{30 \times 1000}{\sqrt{3} \times 13.8}$ = 1255 amps

or on the line side

Base current = $\frac{30 \times 1000}{\sqrt{3} \times 120}$ = 144.3 amps

∴ Fault current = 144.3 × 8.34 = 1203 amps

L-L Fault: Here only positive and negative sequence networks are required.

$$I_{a_1} = \frac{1 + j0.0}{Z_1 + Z_2} = \frac{1 + j0.0}{j0.146 + j0.146} = \frac{1 + j0.0}{j0.292} = -j3.42$$

$$\therefore \qquad I_{a_1} = -I_{a_2} = -j3.424$$

Fault current $\quad I_b = -I_c = \lambda^2 I_{a_1} + \lambda I_{a_2}$ as $I_{a_0} = 0$

$$I_b\,(-0.5 - j0.866)(-j3.424) + (-0.5 + j0.866)(j3.424)$$

$$= j1.712 - 2.965 - j1.712 - 2.965 = 5.9315 \text{ p.u.}$$

∴ Fault current = 5.9315 × 144.3 = 855.9 amps

L-L-G Fault: Here

$$I_{a_1} = \frac{1 + j0.0}{j0.146 + \dfrac{j0.146 \times j0.06767}{j0.146 + j0.06767}} = \frac{1 + j0.0}{j0.19224} = -j5.2 \text{ p.u.}$$

$$I_{a_2} = -\frac{I_{a_1} Z_0}{Z_2 + Z_0} = \frac{+j5.2 \times j0.06767}{j0.21367} = j1.647$$

$$I_{a_0} = j3.553$$

The fault current is

$$I_b + I_c = 3I_{a_0} = 3 \times j3.553 \text{ p.u.}$$

∴ The fault current = 3 × 3.553 × 144.3 = 1538 amps.

13.12 REACTORS

Reactor is a coil which has high inductive reactance as compared to its resistance and is used to limit the short circuit current during fault conditions. To perform this function it is essential that magnetic saturation at high current does not reduce the coil reactance. If an iron cored inductor is expected to maintain constant reactance for currents two to three times its normal value it will turn out to be very costly and heavy. Therefore air cored coils having constant inductance are generally used for current limiting reactors.

Air cored reactors are normally of two types: (*i*) oil immersed type, and (*ii*) dry type. Oil immersed reactors can be cooled by any of the means used for cooling the power transformer whereas the dry type are usually cooled by natural ventilation and are sometimes designed with forced-air and heat exchanger auxiliaries. Reactors are usually built as single phase units.

With the increase in interconnection of power system the fault levels are increasing. It is, therefore, necessary to increase the reactance by introducing reactors at strategic points in the system. The following are the various possibilities:

(*i*) *Generator Reactors*: The reactance of modern alternators may be as high as 2.0 p.u. which means even a dead short-circuit at the terminals of the alternator will result in a current less than full load current and, therefore, no external reactor is required for limiting the short circuit current of such a machine. However, if some old machines are being used alongwith the modern alternator, these old machines need the reactors for limiting the short circuit current. The location of reactors is given in Fig. 13.24(*a*).

(*ii*) *Feeder Reactor*: The per unit value of reactance of a feeder based on its ratings may be small but when compared with the rating of the whole system, its value is quite large and hence a small reactor will be effective in limiting the short circuit current should a fault occur close to the generating station. In case this feeder reactor is not there, a fault in such a location would bring the bus bar voltage almost down to zero value and there is a possibility of various generators falling out of step. We know that, to improve the transient stability of a system the critical clearing angle should be as small as possible, *i.e.*, the breakers should be as fast as possible. In order to obtain this situation and at the same time to reduce the current to be interrupted the feeder must be associated with a reactor (Fig. 13.24(*b*)).

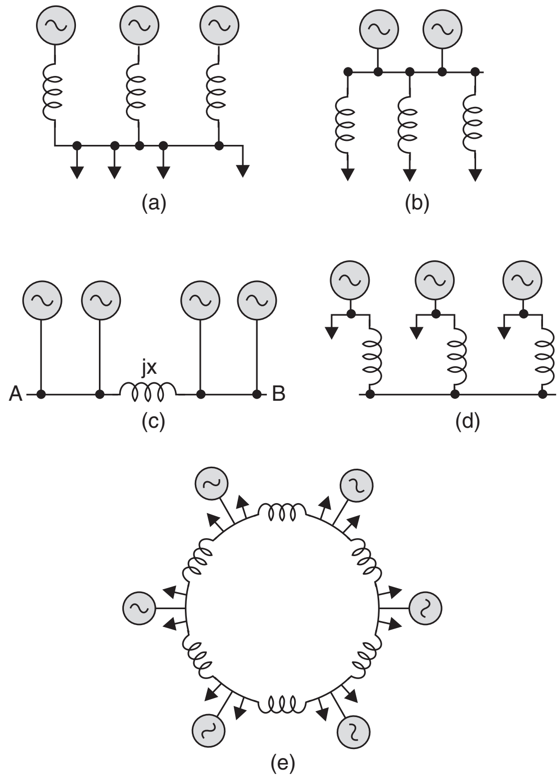


Fig. 13.24 Types of reactors: (*a*) Generator; (*b*) Feeder and (c-e) busbar.

(*iii*) *Busbar reactor*: There are three methods of interconnecting the busbar through the reactors as shown in Fig. 13.24 (*c-e*). The simple method is suitable for plants of moderate output whereas for large-sized plants either the star or ring system of connection is used. It is to be noted that any transfer of power from say section A to section B of the generators, a difference in potential between the bus section is developed. If the power to be transferred is wattless the difference in voltage between the bus section will be much more as compared to when active power of same magnitude will be transferred. Refer to phasor diagram (Fig. 13.25) for the two conditions when resistance of the system is neglected.

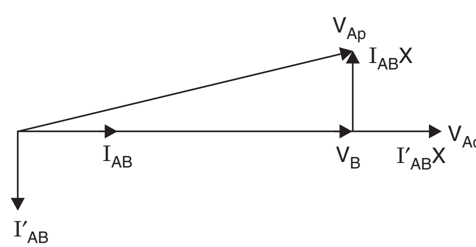


Fig. 13.25 Phasor diagram for Fig. 13.24(*b*).

V_{Ap} is the voltage of bus A when active power is transferred and V_{Aq} is the voltage of bus A when reactive power of same magnitude is transferred from A to B. Since the allowable voltage difference between the bus sections is quite limited it is desirable to meet the wattless requirement of load at bus B by adjusting excitation of the plant at B and the active power requirement can be met by transferring power from A to B.

In case of the ring arrangement, the current to be transferred between two sections flows through two paths in parallel whereas in tie-bar or star system the current flows through two reactors in series. As a result of this configuration whenever a busbar connection is removed for repairs or maintenance in case of a ring arrangement, the maximum power that can be transferred reduces materially which is not the case in case of tie-bar system. For protection the two arrangements involve almost the same cost, except in the limit, it is advantageous to use the tie-bar system.

Calculation of 3-phase Short-Circuit Currents

The sudden short-circuit of a 3-phase alternator has been discussed in Chapter 12. It is shown there that the impedance of the alternator grows from the instant of short circuit to the steady state condition. Which impedance should be considered for evaluating the short-circuit currents, depends upon whether subtransient, transient or steady state short circuit current is required.

The p.u. impedance of an equipment = $\dfrac{IZ}{V}$

where Z is the impedance of the equipment in ohms and I and V are the rated current and voltage respectively.

Now $$I_{sc} = V/Z$$

$$\therefore \quad Z_{\text{p.u.}} = \frac{IZ}{V} = \frac{I}{I_{sc}} = \frac{IV}{I_{sc}V}$$

If VI is the base or full load volt-amperes and VI_{sc} the short-circuit volt-amperes, then

$$Z_{\text{p.u.}} = \frac{\text{Base or full load volt-amperes}}{\text{Short-circuit volt-amperes}}$$

or $$\text{S.C. MVA} = \frac{\text{Base or full load MVA}}{Z_{\text{p.u.}}}$$

This is the relation that will be used for evaluating the short circuit MVA.

13.13 CONCEPT OF SHORT-CIRCUIT CAPACITY OF A BUS

Consider Fig. 13.26. The diagram shown is a part of a large interconnected system. Assume that a symmetrical short circuit occurs at bus 1.

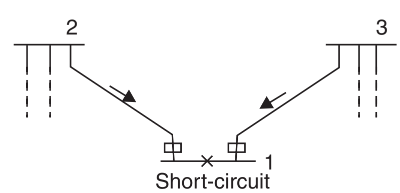


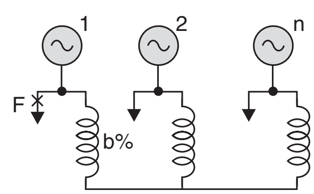
Fig. 13.26 A three-bus system with short-circuit at bus 1.

The prefault voltage of bus is 1 p.u. and as soon as the fault takes place, the voltage of this bus reduces to almost zero. The voltage of the other buses will sag during the short-circuit and the reduction in voltage of various buses is an indication of the "strength" of the network. We normally are interested in knowing this strength and the severity of the short-circuit stresses. Both these objectives are met by a quantity known as short-circuit capacity or fault level of the bus in question. By strength of a bus is meant the ability of the bus to maintain its voltage when a fault takes place at other bus. Of course when a fault takes place at the bus in question, the voltage of this bus will reduce to zero but in case a fault takes place at some other bus then how far the bus in question is able to maintain its voltage is a measure of the strength of the bus. The short-circuit capacity is defined as the product of the magnitude of prefault voltage and post-fault current. Since the strength of a bus is directly related to its short-circuit capacity, the higher the short circuit capacity of the bus the more it is able to maintain its voltage in case of a fault on any other bus. Also it can be seen that higher the short-circuit capacity, lower will be the equivalent impedance as seen between the faulted bus and the zero potential bus of the system. For a bus which is infinitely strong or which has infinite short-circuit capacity will have zero equivalent impedance. In fact such a bus is known as "infinite bus". Such a bus is characterized by a zero equivalent impedance and it is able to maintain constant voltage irrespective of where the short circuit takes place except, of course, for a short circuit on the bus itself, when its voltage will reduce to zero.

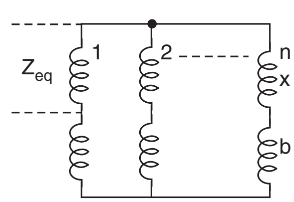
Whenever a short circuit takes place at a bus with higher short-circuit capacity or fault level, high current flows in the bus. This taxes the circuit breaker. The short-circuit stress to which a breaker is subjected is directly related to short-circuit capacity rather than the short-circuit current for two reasons. The first job of the breaker is to extinguish the short-circuit current and once it has extinguished the arc, the breaker contacts must maintain sufficient insulation strength to withstand the voltage (recovery voltage) that appears across them. Since the recovery voltage is 1 p.u. it is logical to rate a breaker for both the post-fault current and prefault voltage, *i.e.*, in terms of short-circuit capacity rather than the short-circuit current.

Example 13.7: A generating station having n section busbars each rated at Q kVA with $x\%$ reactance is connected on the tie-bars system through busbar reactances of $b\%$. Determine the short-circuit kVA if a 3-phase fault takes place on one section. Determine the short-circuit kVA when n is very large.

Solution: The tie-bar system is represented as follows:



Let the fault take place at F. The equivalent circuit will be as follows:



The equivalent impedance Z_{eq} between the zero potential bus and the fault point is

$$\left\{\frac{b+x}{n-1}+b\right\} \parallel x \quad \text{or} \quad \frac{bn+x}{n-1} \parallel x$$

or
$$\frac{1}{Z_{eq}} = \frac{1}{x} + \frac{(n-1)}{(bn+x)}$$

$\therefore$ The short-circuit kVA = $\frac{Q}{Z_{eq}} \times 100 = Q\left[\frac{1}{x} + \frac{(n-1)}{bn+x}\right] \times 100$

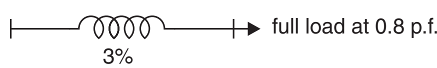
Now, if n is very large,

$$Q\left[\frac{1}{x} + \frac{1-1/n}{b+x/n}\right] = Q\left[\frac{1}{x} + \frac{1}{b}\right]$$

The short-circuit MVA is independent of the number of section. This is the main advantage of tie-bar system. This effectively means that any extension of a large tie-bar interconnected system will not require replacement of the existing switchgear system.

Example 13.8: Determine the percentage increase of busbar voltage required to compensate for the reactance drop when the feeder having a reactance of 3% carries a full load current at a p.f. 0.8 lagging.

Solution: The system is shown below:



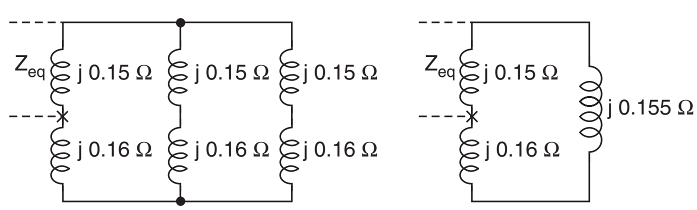
For a series impedance the approximate % drop in volts = $v_r \cos \phi_r + v_x \sin \phi_r$, where v_r and v_x are the per cent resistance and reactance of the series element respectively. Since the feeder has negligible resistance $v_r = 0$.

$\therefore$ Per cent drop of volts = $v_x \sin \phi_r = 3 \times 0.6 = 1.8\%$. **Ans.**

Example 13.9: A small generating station has a busbar divided into three sections. Each section is connected to a tie-bar with reactors each rated at 5 MVA, 0.1 p.u. reactance. A generator of 8 MVA rating and 0.15 p.u. reactance is connected to each section of the busbar. Determine the short-circuit capacity of the breaker if a 3-phase fault takes place on one of the sections of busbar.

Solution: Let the base MVA be 8 MVA, the per unit reactance of the generator be 0.15 p.u. and that of the reactor 0.1 × 8/5 = 0.16 p.u.

The equivalent circuit is as shown below:



The equivalent impedance $Z_{eq} = \frac{j0.15 \times j0.315}{j0.465} = j0.1016129$

$\therefore$ Short-circuit capacity = $\frac{\text{Base MVA}}{Z_{eq}} = \frac{8}{j0.1016129}$ = 78.73 MVA.

Example 13.10: Two generating stations having short-circuit capacities of 1200 MVA and 800 MVA respectively and operating at 11 kV are linked by an interconnected cable having a reactance of 0.5 ohm per phase. Determine the short-circuit capacity of each station.

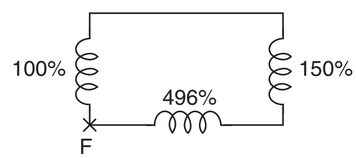
Solution: Assuming base MVA as 1200, the per cent reactance of one generating station is 100% and that of the other is

$$\frac{1200}{800} \times 100 = 150\%$$

The % reactance of the cable is

$$\frac{0.5 \times 1200}{11 \times 11} \times 100 = 496\%$$

When a 3-phase fault takes place at 1200 MVA capacity plant the equivalent circuit will be as follows:

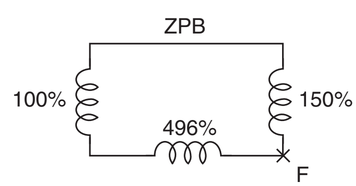


When the fault is at F, fault impedance between F and the neutral bus will be 86.59%.

∴ The short-circuit MVA of this bus will be as follows:

$$\frac{1200}{86.59} \times 100 = 1386 \text{ MVA.} \quad \textbf{Ans.}$$

For fault at the other station, the equivalent circuit will be as follows:



The equivalent fault impedance between F and neutral bus will be 119.84%.

∴ The short-circuit MVA will be

$$\frac{1200}{119.84} \times 100 = 1001 \text{ MVA.} \quad \textbf{Ans.}$$

Example 13.11: Determine the fault MVA, if a fault takes place at F in the diagram shown (Fig. E.13.11). The p.u. values of reactance are given with 100 MVA as base. Resistance may be neglected.

In order to draw Fig. E.13.11(c) from (b), we draw two buses neutral N and the fault point bus F and arrange the various elements of (b) between these buses. The other network reductions are quite clear from the figures till we arrive at (g), where the equivalent fault impedance between the neutral bus and the fault point is 0.14 p.u.

$$\therefore \qquad \text{The S.C. MVA} = \frac{100}{0.14} = 714.28 \text{ MVA.} \quad \textbf{Ans.}$$

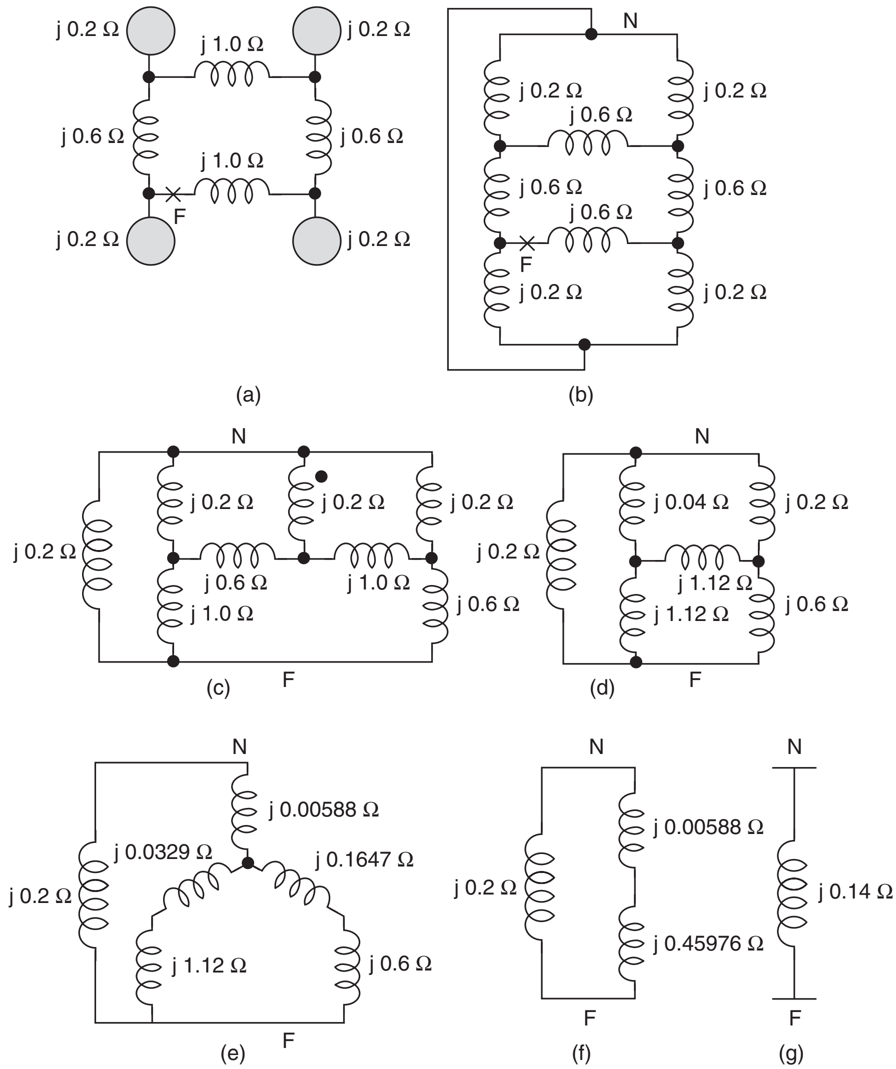


Fig. E.13.11

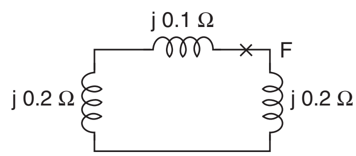
Example 13.12: An alternator and a synchronous motor each rated for 50 MVA, 13.2 kV having subtransient reactance of 20% are connected through a transmission link of reactance 10% on the base of machine ratings. The motor acts as a load of 30 MW at 0.8 p.f. lead and terminal voltage 12.5 kV when a 3-phase fault takes place at the motor terminals. Determine the subtransient current in the alternator, the motor and the fault.

Solution: Taking base quantities as 50 MVA, 13.2 kV,

The base current = $\dfrac{50 \times 1000}{\sqrt{3} \times 13.2}$ = 2186 amps

The prefault voltage = $\dfrac{12.5}{13.5}$ = 0.9469 p.u.

Take this voltage as the reference.



The fault impedance = $\dfrac{j0.3 \times j0.2}{j0.5} = j0.12$ p.u.

∴ The fault current = $\dfrac{0.9469}{j0.12} = -j7.89$ p.u.

The full load current before the fault takes place = $\dfrac{30 \times 1000}{\sqrt{3} \times 12.5 \times 0.8}$ = 1732 amps

$$\text{p.u. load current} = \frac{1732}{2186} = 0.7923 \angle 36.8°$$
$$= 0.6344 + j0.4746$$

The p.u. fault current supplied by the motor = $-j7.89 \times 3/5 = -j4.734$

and that supplied by the generator = $-j7.89 \times 2/5 = -j3.156$.

∴ The net current supplied by the generator during fault

$= -j3.156 + 0.6344 + j0.4746$

$= 0.6344 - j2.6814 = 2.755$ p.u.

The net current supplied by the motor $- 0.6344 - j0.4746 - j4.734$

$= (-0.6344 - j5.2086)$ p.u. = 5.247 p.u.

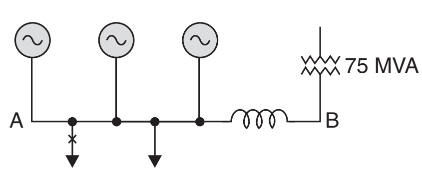
∴ Fault current from the generator = 2.755 × 2186 = 6022 amps.

Fault current from the motor = 5.247 × 2186 = 11470 amps

and fault current = $-j17247$ amps. **Ans.**

Example 13.13: A station operating at 33 kV is divided into sections *A* and *B*. Section *A* consists of three generators 15 MVA each having a reactance of 15% and section *B* is fed from the grid through a 75 MVA transformer of 8% reactance. The circuit breakers have each a rupturing capacity of 750 MVA. Determine the reactance of the reactor to prevent the breakers being overloaded if a symmetrical short circuit occurs on an outgoing feeder connected to *A*.

Solution: The system is given below:

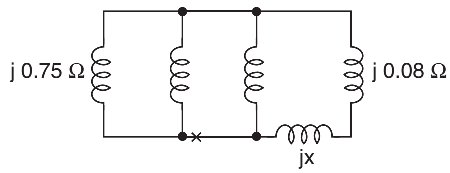


Assume the base MVA as 75.

The p.u. reactance of each generator = $0.15 \times \dfrac{75}{15} = j0.75$

The p.u. reactance of transformer = $j0.08$ p.u.

Let $x\%$ be the reactance of the reactor for base of 75 MVA. The equivalent circuit for a fault of A is as shown in diagram.



The per cent impedance between the fault point and the neutral bus is

$$\frac{0.25(X+0.08)}{0.25+X+0.08} = \frac{0.25X+0.0200}{X+0.33}$$

Now $$\text{S.C. MVA} = \frac{\text{Base MVA}}{\text{p.u. impedance}}$$

or $$750 = \frac{75(X+0.33)}{0.25X+0.02}$$

$$187X + 15.00 = 75X + 24.75$$

$$112X = 9.75$$

$$X = 0.08705 \text{ p.u.}$$

$\therefore$ Actual value of reactance in ohms = $\dfrac{0.08705 \times 33^2}{75}$ = 1.264 ohms. **Ans.**

Example 13.14: A double line to ground fault occurs on phases b and c, at point P in the circuit whose single line diagram is shown in Fig. 13.22(a). Determine the subtransient currents in all phases of machine-1, the fault current and the voltages of machine I and voltages at the fault point. Neglect pre-fault current. Assume that machine-2 is a synchronous motor operating at rated voltage. Both the machines are rated 1.25 MVA, 600 volts with reactances of $X'' = X_2$ = 8% and X_0 = 4%. Each 3-phase transformer is rated 1.25 MVA, 600 volts delta/4160 volts star with leakage reactance of 5%. The reactances of transmission line are $X_1 = X_2$ = 12% and X_0 = 40% on a base of 1.25 MVA, 4160 volts.

Solution: Select 600 volts, 1.25 MVA as base quantities in the generator circuit. Since the transformation ratio is 600/4160 volts and the transformer is rated at 1.25 MVA, no change of reactances is required.

From Fig. 13.22, the Thevenin's equivalent impedances are:

$$Z_{1\,eq} = (8 + 5) \parallel (8 + 5 + 12) = 8.55\%$$

$$Z_{2\,eq} = 8.55\%$$

$$Z_{0\,eq} = 5 \parallel 45 = 4.5\%$$

Now $$I_{a_1} = \frac{E_a}{Z_{1\,eq} + \dfrac{Z_{0\,eq}Z_{2\,eq}}{Z_{0\,eq}+Z_{2\,eq}}}$$

$$I_{a_1} = \frac{1.0}{j0.0855 + \dfrac{j0.0855 \times j0.045}{j0.1305}}$$

$$= -j\,8.697 \text{ p.u.}$$

$$I_{a_2} = \frac{I_{a_1} Z_{0\,eq}}{Z_{0\,eq} + Z_{2\,eq}}$$

$$= -\frac{j8.697 \times j0.045}{j0.1305}$$

$$= j\,3.0 \text{ p.u.}$$

$$I_{a_0} = -\frac{j8.697 \times j0.0855}{j0.1305}$$

$$= j\,5.698 \text{ p.u.}$$

$$V_{a_1} = 1.0 - (-j8.697)\,(j0.0855)$$

$$= 1.0 - 0.7436 = 0.2564$$

$$V_{a_2} = -I_{a_2} Z_{2\,eq} = -(j\,3.0)\,(j\,0.0853)$$

$$= 0.2564$$

Similarly $V_{a_0} = 0.2564$

The fault current $I_a = 0$

$$I_b = (-0.5 - j0.866)\,(-j8.697) + (-0.5 + j0.866)\,(j3.0) + j5.698$$

$$= j4.3485 - 7.5316 - j1.5 - 2.598 + j5.698$$

$$= -10.1296 + j\,8.5465$$

$$= 13.25\underline{/139.85}$$

$$I_c = (-0.5 + j\,0.866)\,(-j8.697) + (-0.5 - j0.866)^*\,(j3.0) + j5.698$$

$$= -10.1296 - j\,8.5465$$

$$= 13.25\underline{/220.15}$$

The current supplied by machine 1 are

$$I_{a_1} = -j8.697 \times \frac{25}{38} = -j5.722$$

or $I_{A_1} = jI_{a_1} = 5.722$

$$I_{a_2} = j3 \times \frac{25}{38} = j1.9737$$

or $I_{A_2} = 1.9737$

∴ $I_A = 5.722 + 1.9737 = 7.6956$

$$I_B = (-0.5 - j0.866)\,(5.722) + (-.05 + j0.866)\,(1.9737)$$

$$= -2.861 - j4.955 - 0.9868 + j1.709$$

$$= -3.8478 - j3.246$$

$$= 5.034\angle 220.15$$

$$I_C = 5.034\angle 139.85$$

Voltages at the fault point

$$V_a = V_{a_1} + V_{a_2} + V_{a_0}$$

$$= 3 \times 0.2564$$

$$= 0.7692 \text{ p.u.}$$

$$V_b = 0 \text{ and } V_c = 0$$

$$V_{ab} = V_a - V_b = 0.2564 - 0 = 0.2564 \text{ p.u.}$$

$$V_{bc} = V_b - V_c = 0.0$$

$$V_{ca} = V_c - V_a = -0.2564 \text{ p.u.}$$

Now the sequence voltages in the generator circuit are

$$V_{A_1} = 1.0 - (5.722)(j0.05)$$

$$= 1 - j0.2861$$

$$V_{A_2} = -1.9737^* (j0.05)$$

$$V_A = 1 - j0.2861 - j0.098685$$

$$= 1 - j0.3848$$

$$= 1.0715\angle 21^\circ \text{ p.u.}$$

$$V_B = (-0.5 - j0.866)(1 - j0.2861) + (-0.5 + j0.866)(-j0.09868)$$

$$= -0.5 + j0.1430 - j0.866 - 0.24776$$

$$= -0.74776 - j0.723$$

$$= 1.04\angle 224$$

$$V_C = (-0.5 + j0.866)(1 - j0.2861) + (-0.5 - j0.866)(-j0.09868)$$

$$= -0.74776 + j0.723$$

$$= 1.04\angle 136^\circ$$

$$V_{AB} = V_A - V_B$$

$$= 1.0 - j0.3848 + 0.74776 + j0.723$$

$$= 1.74776 + j0.3382$$

$$= 1.78\angle 10.95^\circ$$

$$V_{BC} = V_B - V_C$$

$$= -0.74776 - j0.723 + 0.74776 - j0.723$$

$$= -j1.446$$

$$V_{CA} = V_C - V_A$$

$$= -0.74776 - j0.723 - 1 + j0.3848$$

$$= -1.74776 - j0.3382$$

$$= 1.78\angle 190.95 \quad \textbf{Ans.}$$

Example 13.15: A generator supplies a motor through a Y/Δ transformer. The generator is connected to the star side of the transformer. A fault occurs between the motor terminals and the transformer. The symmetrical components of the subtransient current in the motor flowing towards the fault are $I_{a_1} = -0.8 - j2.6$ p.u., $I_{a_2} = -j2.0$ p.u. and $I_{a_0} = -j3.0$ p.u. From the transformer towards the fault $I_{a_1} = 0.8 - j0.4$ p.u., $I_{a_2} = -j1.0$ p.u. and $I_{a_0} = 0$. Assume $X'' = X_2$ for both the motor and the generator. Describe the type of fault. Find (*i*) the pre-fault current if any, in line '*a*' (*ii*) the subtransient fault current in p.u. and (*iii*) the subtransient current in each phase of the generator in p.u.

Solution: The system is shown in Fig. E.13.15.1

Since the currents contain zero sequence components the fault is either *L-G* or *L-L-G*. The total fault current is the sum of fault currents fed from the transformer side and the motor side.

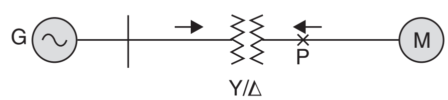


Fig. E.13.15.1

Total positive sequence fault current

$$= -0.8 - j2.6 + 0.8 - j0.4$$
$$= -j3.0$$

Similarly, it is found that total negative sequence and zero sequence fault currents are $I_{a_2} = -j3.0$ and $I_{a_0} = -j3.0$. Since all the three sequence components of current are equal, it is a *L-G* fault.

(*i*) Let the prefault current be $a + jb$ and since for *L-G* fault total I_{a_1} = total I_{a_2} = total $I_{a_0} = -j3.0$ p.u. in the case. The distribution of negative sequence current is $-j2.0$ p.u. from the motor and $-j1.0$ from the generator side *i.e.*, the ratio of the reactance from the two sides is 1 : 2 *i.e.*, it is given as in Fig. E.13.15.2.

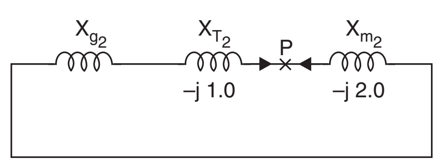


Fig. E.13.15.2

Therefore, positive sequence current supplied by the motor would be $-j2.0$ and that by generator $-j1.0$ if the prefault current is neglected. Now considering the prefault current, we should have positive sequence current supplied by the motor as

$$-j2.0 - (a + jb) = -0.8 - j2.6$$

Separating the real and imaginary quantities, we have $-2 - b = -2.6$

and $\quad -a = -0.8$

or $\quad b = 0.6$

and $\quad a = 0.8$

Therefore, the prefault current is

$$(0.8 + j0.6)$$

(*ii*) The subtransient fault current = $3I_{a_1} = -j9.0$ p.u.

(*iii*) The sequence components from the generator are

$$I_{a_1} = 0.8 - j0.4,\ I_{a_2} = -j1.0,\ I_{a_0} = 0$$

$$I_{A_1} = j(0.8 - j0.4) I_{A_2} = -j(-j1.0)$$

$$I_{A_1} = j0.8 + 0.4 \quad I_{A_2} = -1.0$$

$$I_A = I_{A_1} + I_{A_2} = +j0.8 + 0.4 - 1.0$$

$$= -0.6 + j0.8. \quad \textbf{Ans.}$$

$$I_B = \lambda^2 I_{A_1} + \lambda I_{A_2}$$

$$= (0.5 - j0.866)(j0.8 + 0.4) + (-0.5 + j0.866)(-1.0)$$

$$= -j0.4 - 0.2 + 0.6928 - j0.3464 + 0.5 - j0.866$$

$$= 0.9928 - j1.6124. \quad \textbf{Ans.}$$

$$I_c = \lambda^2 I_{A_1} + \lambda^2 I_{A_2}$$

$$= (-0.5 + j0.866)(0.4 + j0.8) = (-0.5 - j0.866)(-1.0)$$

$$= -0.2 - j0.4 + j0.3464 - 0.6928 + 0.5 + j0.866$$

$$= -0.3928 + j0.8124. \quad \textbf{Ans.}$$

Similarly the currents from the motor side can be computed.

Example 13.16: A transformer is rated at 11 kV/0.4 kV, 500 kVA, 5% reactance. Determine the short circuit MVA of the transformer when connected to an infinite bus.

Solution: Since the transformer is connected to an infinite bus, the p.u. impedance of the circuit will be 0.05 *i.e.*, the p.u. impedance offered by the transformer.

$$\therefore \qquad \text{S.C. MVA} = \frac{0.5}{0.05} = 10 \text{ MVA}. \quad \textbf{Ans.}$$

Example 13.17: Three identical resistors are star connected and rated 2500 volts, 500 kVA as a three phase unit. The resistors are connected to the low-tension side of a Δ/Y transformer. The voltage at the resistor load are

$$|V_{ab}| = 2000 \text{ volts},\ |V_{bc}| = 2800 \text{ volts}$$

and $|V_{ca}| = 2500$ volts. Select base as 2500 volts

500 kVA, find the line voltages and currents in per unit on the delta side of the transformer. Assume that the neutral of the load is not connected to the neutral of the transformer secondary.

Solution: The per unit voltages are

$$|V_{ab}| = \frac{2000}{2500} = 0.8 \text{ p.u.}$$

$$|V_{bc}| = \frac{2800}{2500} = 1.12 \text{ p.u.}$$

$$|V_{ca}| = 1.0 \text{ p.u.}$$

Assuming an angle of 180° of V_{ca} and using the law of cosines.

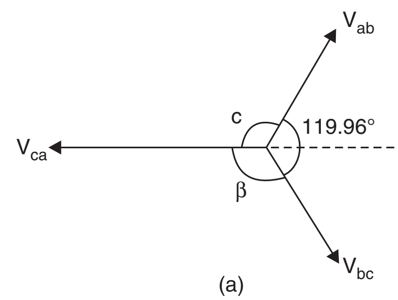


Fig. E.13.17(*a*)

$$1.12^2 = 0.8^2 + 1.0^2 + 2*0.8*1.0 \cos \alpha$$

$$\alpha = 103.94°$$

Similarly

$$0.8^2 = 1.12^2 + 1.0^2 + 2*1*1.12 \cos \beta$$

$$\beta = 136.1°$$

The line voltages are

$\therefore$ $$V_{ab} = 0.8\angle 76.06$$

$$V_{ca} = 1.0\angle 180°$$

$$V_{bc} = 1.12\angle -43.9°$$

The symmetrical components of the line voltages are

$$V_{ab_1} = \frac{1}{3}[V_{ab} + \lambda V_{bc} + \lambda^2 V_{ca}]$$

$$= \frac{1}{3}[0.8\angle 76.06 + 1.12\angle -43.9 + 120 + 1.0\angle 180 + 240°]$$

$$= \frac{1}{3}[0.1927 + j\,0.7764 + 0.2690 + j1.0872 + 0.5 + j\,0.866]$$

$$= 0.3205 + j0.9098$$

$$= 0.9646\angle 70.59°$$

$$V_{ab_2} = \frac{1}{3}[0.8\angle 76.06 + 1.12\angle -43.9 + 240 + 1.0\angle 180 + 120]$$

$$= \frac{1}{3}[0.1927 + j0.7764 - 1.0760 - j0.3106 + 0.5 - j0.866]$$

$$= -0.1277 - j0.1334$$

$$= 0.1846\angle 226.25°$$

As neutral is isolated $V_{ab_0} = 0$.

In order to evaluate the positive and negative sequence components of phase to neutral voltage, we take V_{ab_1} and V_{ab_2} as the reference phases as shown in the following figure.

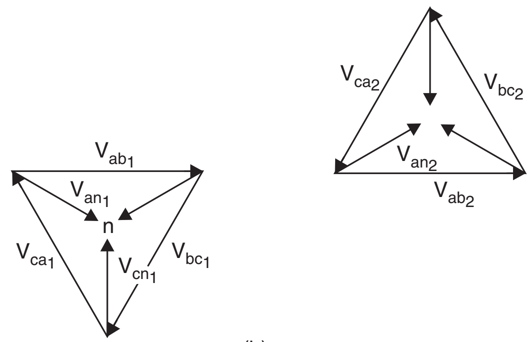


Fig. E.13.17(*b*)

From Fig. 13.17(*b*)

$$V_{an_1} = V_{ab_1} \angle -30° = 0.9646 \angle 70.59 - 30 = 0.9646 \angle 40.59 \text{ p.u.}$$

and

$$V_{an_2} = V_{ab_2} \angle 30° = 0.1846 \angle 226.25 + 30° = 0.1846 \angle 256.25$$

Since each resister has an impedance of $1.0 \angle 0°$ p.u.

$$I_{a_1} = \frac{V_{an1}}{1.0 \angle 0°} = 0.9646 \angle 40.59$$

and

$$I_{a_2} = \frac{V_{an_2}}{1.0 \angle 0°} = 0.1846 \angle 196.25°$$

V_{an_1} and V_{an_2} are the voltages on the star connected low voltage side of the transformer. The corresponding voltages on the delta side (high tension) are

$$V_{A_1} = -jV_{an_1} = 0.9646 \angle 40.59 - 90 = 0.9646 \angle -49.41 = 0.6276 - j0.7325$$

$$V_{A_2} = -jV_{an_2} = 0.1846 \angle 256.25 + 90 = 0.1846 \angle -13.75° = 0.1793 - j\,0.04387$$

$$\therefore \quad V_A = V_{A_1} + V_{A_2} = 0.9646 \angle -49.41 + 0.1846 \angle -13.75° = 0.8069 - j\,0.7763 = 1.12 \angle -43.9$$

$$V_{B_1} = \lambda^2 V_{A_1} = 0.9646 \angle -49.41 + 240 = -0.9481 - j\,0.1773$$

$$V_{B_2} = \lambda V_{A_2} = 0.1846\underline{/-13.75+120^\circ}$$
$$= -0.0516 + j\,0.1772$$
$$V_B = V_{B_1} + V_{B_2} = 1.0\underline{/180^\circ}$$
$$V_{C_1} = \lambda V_{A_1} = 0.9646\underline{/-49.41+120^\circ}$$
$$= 0.3205 + j\,0.9097$$

∴ $$V_{C_2} = \lambda^2 V_{A2} = 0.1846\underline{/-13.75+240^\circ}$$
$$= -0.1276 - j0.1333$$
$$V_C = V_{C_1} + V_{C_2} = 0.1929 + j\,0.7764$$
$$= 0.8\underline{/76.06}$$

Now $$V_{AB} = V_A - V_B$$
$$= 0.8069 - j0.7763 + 1$$
$$= 1.8069 - j\,0.7763$$
$$= 1.967\underline{/-23.25}\ \text{(line to neutral voltage base)}$$
$$= 1.1356 - 23.25\ \text{(line to line voltage base)}$$

Similarly $$V_{BC} = V_B - V_C$$
$$= -1.0 - 0.1929 - j0.7764$$
$$= -1.1929 - j0.7764$$
$$= 1.423\underline{/213.05^\circ}\ \text{p.u. (line to neutral voltage base)}$$
$$= \frac{1.423}{\sqrt{3}}\underline{/213.05}$$
$$= 0.8215\underline{/213.05^\circ}\ \text{p.u. (line to line voltage base)}$$
$$V_{CA} = V_C - V_A$$
$$= 0.1929 + j0.7764 - 0.8069 + j0.7763$$
$$= -0.614 + j1.5527$$
$$= 1.6697\underline{/111.57^\circ}\ \text{(line to neutral voltage base)}$$
$$= 0.9639\ \underline{/111.57^\circ}\ \text{p.u. (line to line voltage base)}$$

As the load impedance in each phase is resistive and one p.u., I_{a_1} and V_{an_1} are found to have identical p.u. values. Similarly I_{a_2} and V_{an_2} are identical in p.u. Therefore, I_A must be identical to V_A expressed in p.u. thus

$$I_A = 1.12\underline{/-43.9}$$
$$I_B = 1.0\ \underline{/180^\circ}$$

and $$I_C = 0.8\underline{/76.06^\circ}.\quad \textbf{Ans.}$$

PROBLEMS

13.1. The line currents in a 3-phase supply to an unbalanced load are respectively $I_a = 10 + j20$, $I_b = 12 - j10$ and $I_c = -3 - j5$ amperes. The phase sequence is *abc*. Determine the sequence components of currents.

13.2. The voltages across a 3-phase unbalanced load are V_a = 300 V, V_b = 300 ∠ – 90° V and V_c = 800 ∠ 143.1° V respectively. Determine the sequence components of voltages. Phase sequence is *abc*.

13.3. Three 6.6 kV, 12 MVA, 3-phase alternators are connected to a common set of busbars. The positive, negative and zero sequence impedances of each alternator are 15%, 12% and 4.5% respectively. If an earth fault occurs on one busbar, determine the fault current:

(*i*) if all the alternator neutrals are solidly grounded;

(*ii*) if one only of the alternator neutrals is solidly earthed and the others are isolated;

(*iii*) if one of the alternator neutrals is earthed through a reactance of 0.5 ohm and the others are isolated.

13.4. A 3-phase alternator is connected to a star/delta transformer through a transmission line as shown here:

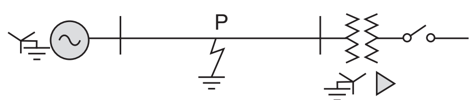


Fig. P.13.4

The positive, negative and zero sequence impedances of the alternator are *j*0.1, *j*0.1 and *j*0.05 p.u. respectively and those of transformer are *j*0.05 p.u. each. A line-to-ground fault occurs at *P* as shown. The respective sequence impedances on the left and right of the fault point are $X''_L = j0.2$ p.u., $X_{L_2} = j0.2$ p.u. and $X_{L_0} = j0.4$ p.u. and $X_r'' = j0.2$, $X_{r_2} = j0.2$ and $X_{r_0} = j0.4$ p.u. Determine the line current feeding into the fault and voltages at the fault when (*i*) the generator is grounded as shown, (*ii*) the generator neutral is isolated.

13.5. A 50 Hz, 50 MVA, 13.2 kV star grounded alternator is connected to a line through a Δ/*Y* transformer as shown here. The positive, negative and zero sequence impedances of the alternator are *j*0.1, *j*0.1 and *j*0.05 p.u. respectively.

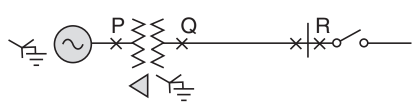


Fig. P.13.5

The transformer rated at 13.2 kV Δ/120 kV *Y*, 50 MVA with star solidly grounded has the sequence impedances of $X'' = X_2 = X_0 = j0.1$ p.u. each. The line impedances between *Q* and *R* are $X'' = j0.03$, $X_2 = j0.03$ and $X_0 = j0.09$ p.u., respectively. Assuming the fault to take place at *P*, determine the subtransient fault current for a (*i*) 3-phase fault, (*ii*) a line-to-ground fault, (*iii*) a line-to-line fault, (*iv*) a double line-to-ground fault. Also express these fault currents as a percentage of 3-phase fault current as calculated in (*i*).

13.6. Solve Problem 13.5 when fault is at point *Q*.

13.7. Solve Problem 13.5 when fault is at point *R*.

13.8. Solve Problem 13.5 when the neutral of the alternator is grounded through an impedance of *j*0.2 ohm.

13.9. A 50 Hz, 13.2 kV, 15 MVA alternator has $X'' = X_2$ = 20% and X_0 = 8% and its neutral is grounded through a reactor of 0.5 ohm. Determine the initial symmetrical r.m.s. current in the ground and in line *c*, when a double line-to-ground fault occurs on phase *b* and *c* and the generator voltage is 12 kV before the fault takes place.

13.10. A 3-phase generator is rated for 60 MVA, 6.9 kV and subtransient reactance $X_d'' = j0.15$ p.u. The generator feeds a motor through a line with impedance of $j0.1$ p.u. on generator rating. The motor is rated at 10 MVA and 6.9 kV with $X_d'' = j0.2$ p.u. on the motor base. The voltage at the terminal of the motor is 1 p.u. and takes a load current of 1.0 p.u. at unity p.f. A symmetrical fault occurs at the motor terminals. Determine the subtransient r.m.s. current at the fault, in the generator and in the motor.

13.11. A 65-MVA star connected 16 kV synchronous generator is connected to a 20 kV/120 kV, 75 MVA Δ/Y transformer. The subtransient reactance X_d'' of the machine is 0.12 p.u. and the reactance of transformer is 0.1 p.u. When the machine is unloaded, a 3-phase fault takes place on the HT side of the transformer. Determine (*i*) the subtransient symmetrical fault current on both sides of the transformer, (*ii*) the maximum value possible of the d.c. current. Assume 1 p.u. generator voltage.

13.12. If in problem 13.11 a 3-phase balanced impedance (on a base of 120 kV and 75 MVA) of $(0.8 + j0.6)$ p.u. ohm is connected across the transformer terminals at 120 kV and a fault takes place beyond the load terminals as shown in Fig. P.13.12, determine the subtransient fault current and the generator current using the Thevenin's theorem. Assume per-fault voltage to be 1.0 p.u.

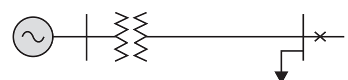


Fig. P.13.12

13.13. Four 50 MVA generators of 15% reactance each are connected via four 35 MVA reactors each of 10% reactance to a common bus bar. The feeders are each connected to the junction of each alternator and its reactor. Determine the rating of each feeder circuit breaker.

13.14. Two 50 MVA, 50 Hz, 11 kV alternators with sub-transient reactance $X'' = j0.1$ p.u. and a transformer of 40 MVA 11 kV/66 kV and reactance of 0.08 p.u. are connected to a bus A. Another generator 60 MVA, 11 kV alternator with reactance of 0.12 p.u. is connected to bus B. Bus A and B are interconnected through a reactor of 80 MVA 20 per cent reactance. If a 3-phase fault occurs on the high voltage side of the transformer, calculate the current fed into the fault.

13.15. Two generating stations having short circuit capacities of 1500 MVA and 1000 MVA respectively and operating at 11 kV are linked by an interconnected cable having a reactance of 0.6 ohm per phase, determine the short circuit capacity of each station.

13.16. A 33 kV 3-phase transmission line of resistance 2 ohm and reactance 8 ohm is connected at each end to 2 MVA 33/6.6 kV Δ/Y transformer. The resistance and reactance drops of the transformers are 1% and 3% respectively. Determine the fault current in each section of the system when a 3-phase fault take place on the low voltage side of the step-down transformer. Assume a source with zero impedance.

13.17. Four busbar sections have each a generator of 40 MVA 10% reactance and a busbar reactor of 8% reactance. Determine the maximum MVA fed into a fault on any bus bar section and also the maximum MVA if the number of similar bus bars in sections is very large.

13.18. A 30 MVA, 11 kV generator has subtransient reactance of 10%, supplies power to three identical motors through a transformer as shown in Fig. P.13.18. Each motor is rated for 8 MVA, 6.6 kV with subtransient and transient reactances of $j0.15$ and $j0.25$ p.u. respectively. The transformer is rated for 30 MVA, 11 kV/6.6 kV and leakage reactance 8%. The motor bus bar voltage is 6.6 kV when a 3-phase fault takes place at F. Determine (*i*) the subtransient current in the fault, (*ii*) the subtransient current in breaker B, (*iii*) the momentary current in breaker B, and (*iv*) the current to be interrupted by breaker B in 8 cycles.

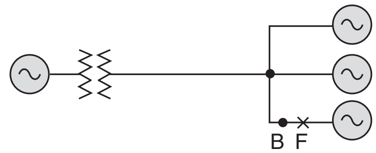


Fig. P.13.18

13.19. A power plant has two generators of 10 MVA, 15% reactance each and two 5 MVA generators of 10% reactance paralleled at a common bus bar from which load is taken through a number of 4 MVA step up transformers each having a reactance of 5%. Determine the short circuit capacity of the breakers on the (*i*) low voltage, and (*ii*) high voltage side of the transformer.

13.20. A 3-phase, 5 MVA, 6.6 kV alternator with a reactance of 8% is connected to a transmission line of series impedance (0.12 + *j*0.48) ohm per km. The transformer is rated at 3 MVA, 6.6 kV/33 kV and reactance 5%. Determine the fault current supplied by the generator operating under no load with voltage 6.9 kV when a 3-phase delta connected fault occurs 15 km along the line with fault impedance between each line being (12 + *j*48) ohms.

13.21. A single line-to-ground fault occurs on phase *a* at point *P* in the circuit whose single line diagram is shown here. Determine the subtransient current in phase *a* of machine 1 and in the fault at *P*. Neglect prefault current. Assume that machine 2 is a synchronous motor operating at rated voltage. Both machines are rated. 1.5 MVA, 600 volts with reactances of $X'' = X_2 = 8\%$ and $X_0 = 4\%$. Each 3-phase transformer is rated 1.25 MVA, 660 volts delta/ 4160 volts star with leakage reactance of 5%. The reactances of transmission line are $X_1 = X_2 = 12\%$ and $X_0 = 40\%$ on a base of 1.25 MVA, 4160 volts.

Fig. P.13.21

13.22. Solve Problem 13.21 when fault is at *R*.

13.23. A 50 Hz, 80 MVA, 11 kV generator has positive, negative and zero sequence impedances of *j*0.4, *j*0.3 and *j*0.1 p.u. respectively. The generator is connected to a busbar *A* through a transformer having $X_1 = X_2 = X_0 = j0.4$ p.u. on 100 MVA base and rated voltage. Determine the ohmic resistance and rating of the earthing resistor such that for a *L-G* fault on busbar *B* the fault current of the generator does not exceed full load current. A reactor of reactance 0.08 p.u. on 100 MVA base is connected between busbars *A* and *B*.

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